At the joint sessions on Wednesday morning, Professor E. T. BELL represented the Association and contributed an interesting and instructive paper on "Successive generalizations in the theory of numbers." This paper will be published in full in the MONTHLY. The other paper was given by Professor E. W. CHITTENDEN, representing the Society, on "The metrization problem and related problems in the theory of abstract sets." Vice-president Evans of the Society presided to introduce Professor Chittenden and President Jackson of the Association to introduce Professor Bell. At this session the following joint resolution was presented and carried by unanimous rising vote:

Resolved, That the American Mathematical Society and the Mathematical Association of America, in joint session at Columbus on September 8, 1926, express their deep sense of loss in the death of Professor Bohannan, on June 20, 1926, and their profound regret that the meetings on the campus so long animated by his personality could not be graced by his presence; and direct that copies of this resolution be transmitted to Mrs. Bohannan and to the President of the Ohio State University.

At the two sessions of the Association on Tuesday the following papers were presented; abstracts of the papers are given, numbered in accordance with the numbers of the papers:

(1) "An outline of the history and applications of the calculus of residues," by Professor W. B. FORD, University of Michigan.

(2) "Waring's problem: A chapter in the additive theory of numbers," by Professor A. J. KEMPNER, University of Colorado.

(3) "Sets of linear equations in positive unknowns," by Professor C. F. GUMMER, Queen's University.

(4) "A general operational analysis," by Mr. W. O. PENNELL, Chief Engineer, Southwestern Bell Telephone Company, St. Louis, Mo.

(5) "On the value of mathematical figures and models," by Professor ARNOLD EMCH, University of Illinois.

(6) "Mathematics in modern engineering practice," by Dr. TOBIAS DANTZIG, Baltimore, Md. (by invitation).

1. The calculus of residues as developed by Cauchy and extensively applied by him to the evaluation of definite integrals and the study of infinite series was outlined first in Professor Ford's paper. Its further applications to the following fields were then briefly discussed, chiefly by means of examples: (*a*) The determination of the remainder in the Maclaurin sum formula, (*b*) the study of the convergence of Fourier series and other allied developments of mathematical physics, (*c*) the problem of the analytic continuation of a function of a complex variable, (*d*) the determination of asymptotic developments, (*e*) applications to the theory of numbers.

2. Professor Kempner's paper will appear in an early issue of the MONTHLY.

3. A system of linear homogeneous equations in which the number of unknowns exceeds the rank of the matrix has a solution involving one or more undetermined quantities. This indeterminate solution may or may not include solutions in which all the unknowns have positive values. A system may be called *simple* if the number of unknowns is one more than the rank. The solution is then determinate except for a common factor; so that the existence of positive solutions is proved or disproved by the signs of the determinants appearing in the ordinary solution formula.

If a system in general has positive solutions it is shown that a *simple* system with positive solutions may be obtained by *abbreviating* the given system, that is, by leaving out the terms in one or more unknowns. In case such a simple abbreviated system has a matrix of rank less than that of the given system, other simple systems may be obtained involving fresh unknowns so that the matrix of coefficients has a greater rank.

It is then found that *a necessary and sufficient condition for the existence of positive solutions is that there exist simple systems with positive solutions whose combined matrix has the given rank.*

The result is extended to finite systems of equations with a denumerably infinite set of unknowns, and also to the continuously infinite case where the sums of terms are replaced by definite integrals. A geometrical interpretation is possible throughout. A test similar to the foregoing applies when only some of the unknowns are required to be positive.

4. Mr. Pennell's paper formed an extension of his paper which appeared in the MONTHLY (*1926*, 293–307).

5. In his paper on the value of mathematical models and figures, which was illustrated by sixty screen projections, Professor Emch pointed out that the reason for choosing the title for his paper was not to put up a defense for the use of figures and models in mathematical instruction, but to show the possibilities of visualization in certain lines of more advanced mathematical teaching and research.

It happens not unfrequently that after the effective construction of a model or of a figure, a close examination of the finished product reveals or suggests the existence of new properties of the forms investigated, not anticipated before the construction. Another important factor of such work lies in the strengthening of the geometrical imagination and mathematical intuition in general. The use of physical images of certain mathematical forms and relations is an incomparable aid in fortifying mathematical memory. The acquisition of a concrete picture of a certain theory, or of certain forms, is a powerful means of mathematical orientation. As a matter of fact, mathematical symbols and formulas must be classed in this category of concrete pictures of mathematical

relations. Geometrical figures and models serve, in a general way, the same purpose. They are merely heuristic auxiliaries and not means in themselves.

According to Professor Emch the large number of projections which might be increased indefinitely should prove convincing enough to show that mathematical visualization even in more advanced fields is worthy of serious consideration.

6. Dr. Dantzig discussed in general the possible field of application of mathematics to engineering and the practical difficulties which are hampering a wider application of mathematical methods to problems of industry. A few incidents and observations based on the experience of the author as a research and consulting engineer were related to substantiate his conclusions and recommendations, which have for their object the furthering of mathematical methods in the field of engineering.

The fundamental problem of strength in the design of machines and structures was taken as an illustration and the parallel development of the theory of elasticity and that of strength of materials was traced since the time of Galileo. Such illustrious mathematicians as Euler, Cauchy, Poisson, and Clebsch developed powerful methods and solved completely a number of important cases, after reducing the general problem to the solution of a system of partial differential equations under given boundary conditions.

The author then discussed the reason why the engineer has failed to take full advantage of the remarkable results achieved by the theory of elasticity and came to the conclusion that some of the difficulties are insurmountable and are inherent in the nature of mathematical methods. Mathematics is probably destined to keep centuries ahead of its practical applications. On the other hand educational institutions and professional organizations can do a great deal in clearing up the minor obstacles. The author made the following recommendations based on his observations and contacts with the practical engineer.

Regarding the mathematics curriculum in colleges and technical schools the author was opposed to the presenting of the subject as a mere weapon in attacking technical problems without regard to the cultural value of mathematics. However, a special course in "Mathematical topics in engineering" would be of tremendous value to the prospective engineer.

The establishment of correspondence and extension courses in mathematics was another step recommended by the author to assist the technical man to a deeper understanding of the mathematical principles underlying his work and in reënforcing and reconstructing his mathematical education.

Finally one of the departments of such an organ as the MONTHLY could be made to serve as a clearing house for technical problems susceptible of mathematical treatment thus broadening the influence of the Association and placing the benefits of mathematical methods at the disposal of wider circles.

The American Mathematical Society held sessions on Wednesday and Thursday for the reading of papers. Seventy-one papers, besides that on the joint program, were read at these sessions, or presented by title.

On Wednesday afternoon Mr. C. H. SINGER of The Banta Publishing Co. and Professor E. R. HEDRICK spoke of the possibilities of the monotype machine for the setting of mathematical articles. The various items of information as to what sorts of notation are readily available and what sorts are particularly expensive merit a full presentation in our mathematical journals, so that they may be available to all who may prepare papers for printing.

The members of Pi Mu Epsilon met at a luncheon Wednesday noon in the private dining room at the Faculty Club.

BUSINESS MEETING OF THE ASSOCIATION

At the business meeting on Tuesday afternoon, called through the MONTHLY, the amendment proposed by the Trustees and printed in the April issue of the MONTHLY was carried by a vote of 40 to 18, after considerable discussion of both sides of the question. By this action Article III, Section 3, of the By-Laws now reads: "The President shall be elected by the Association's members biennially for a term of two years and shall be ineligible for re-election. The Vice-Presidents shall be elected by the Association's members annually, etc."

BUSINESS MEETING OF THE TRUSTEES

Eight members of the Board were present at this session.

The following sixty-four persons, on application duly certified, were elected to individual membership:

H. C. BARBER, A.M. (Amherst). Head of Dept., Charlestown High School, Boston, Mass.

THEODORE BENNETT, Ph.D. (Illinois). Instr., Univ. of Ill., Urbana, Ill.

W. J. BERRY, M.S. (Colorado). Jr. Physicist, U. S. Bureau of Standards, Washington, D. C.

JEANETTE BICKFORD, A.M. (Radcliffe). Head of Dept., High School, Framingham, Mass.

J. H. BINNEY, A.M. (Texas). Asst. Prof, A. and M. Coll. of Texas, College Station, Texas.

L. W. BLAU, A.B. (W.T.S.T.C.). Tutor, Physics, Univ. of Texas, Austin, Texas.

W. M. BORGMAN, JR., B.S. (Michigan). Instr., Coll. of the City of Detroit, Detroit, Mich.

M. LUCILE BRISTOL, Geom. and Alg., Derham Hall High School, St. Paul, Minn.

J. G. CHANEY, A.B. (Southwestern). Instr., High School, Lubbock, Texas.

A. L. CHRISTIAN, A.B. (Baylor). Penelope, Texas.

P. T. COPP, A.M. (Ohio State). Prof., Univ. of Detroit, Detroit, Mich.

J. W. DAVIS, A.M. (Yale). Jr. Master, Dorchester High School for Boys, Dorchester, Mass.

V. C. D'UNGER, B.S. (Little Rock). Instr., Little Rock Coll., Little Rock, Ark.

L. A. DYE, A.M. (Rochester). Instr., Univ. of Rochester, Rochester, N. Y.

D. C. EIPPER, A.B. (Harvard). Head of Dept., Math. and Physics, Berkshire School, Sheffield, Mass.

C. H. FISCHER, B.S. (Washington Univ.). Instr., Beloit Coll., Beloit, Wis.

GEORGE FRECHEVILLE, M.A. (Cantab.). Research, Agric. Economics Research Inst., Univ. of Oxford, Oxford, Eng.

RAYMOND GARVER, A. M. (Montana). Instr., Univ. of Rochester, Rochester, N. Y.

L. M. GRAVES, Ph.D. (Chicago). Asst. Prof., Univ. of Chicago, Chicago, Ill.

L. D. HAERTTER, University High School, Univ. of Minn., Minneapolis, Minn.

R. G. HEIN, A.B. (Wisconsin). Teacher, High School, Waupun, Wis.

A. F. HENDRICKS, B.S. (Valparaiso). Pres. Emeritus, Will Mayfield Coll., Marble Hill, Mo.

ETTA M. HENRY, B.S. (Michigan). Teacher, Ceredo-Kenova High School, Kenova, W. Va.

W. C. HOLBROOK, N. E. Sales Mgr., Henry Holt & Co., Boston, Mass.

C. F. HOLMES, M.S. (Howard). Instr., State Normal School, Elizabeth City, N. C.

J. C. HOSHAUER, B.S. (Bucknell). Head of Dept., High School, Williamsport, Pa.

E. C. KENNEDY, E.M. (Texas). Austin, Texas.

E. C. KLIPPLE, A.B. (Texas). Instr., Pure Math., Univ. of Texas, Austin, Texas.

C. C. LAO, A.B. (Yale-in-China). Instr., Yale-in-China, Changsha, China.

C. L. LEIPER, Grad. (U.S.N.A.). Prof., U. S. Naval Acad., Annapolis, Md.

F. A. LEWIS, A.M. (Alabama). Asso. Prof., Univ. of Ala., University, Ala.

J. V. LONGENECKER, M.S. (Iowa). Actuary, Farmers and Bankers Life Ins. Co., Wichita, Kans.

HENRY LUCEY, A.B. (Amherst). Head of Dept., Jr. and Sr. High Schools, Pittsfield, Mass.

C. T. MALE, M.C.E. (Union). Instr., Union Coll., Schenectady, N. Y.

J. A. MARSH, A.M. (Amherst). Head of Dept., High School of Commerce, Boston, Mass.

W. I. MILLER, A.M. (Bucknell). Fellow, Univ. of Pittsburgh, Pittsburgh, Pa.

H. F. MITCHELL, B.S. (Chicago). Tulsa, Okla.

W. D. MORGAN, St. Paul, Minn.

A. F. MOURSUND, A.B. (Texas). Instr., High School, Austin, Texas.

C. A. MURRAY, A.M. (Texas). Instr., Stephen F. Austin State Teachers Coll., Nacogdoches, Texas.

G. F. NELSON, A.M. (Texas). Teacher, Math. and Physics, High School, Sour Lake, Texas.

C. V. NEWSOM, A.B. (College of Emporia). Head of Dept., High School, Salina, Kans.

F. E. NEWTON, Ph.B. (Yale). Head of Dept., Phillips Acad., Andover, Mass.

EDITH M. NUTT, A.B. (Wellesley). Teacher, Math. and German, High School, Natick, Mass.

L. J. PARADISO, A.M. (Ohio State). Instr., Lehigh Univ., Bethlehem, Pa.

JONATHAN PICKETT, Ranger, Texas.

R. B. PLYMALE, A.M. (Mercer). Instr., Freshman Math., Mercer Univ., Macon, Ga.

J. E. POWELL, M.S. (Ohio State). Instr., Mich. Agric. Coll., East Lansing, Mich.

WALTER ROBERTS, B.S. (Pennsylvania). Head of Dept., West Phila. High School, Philadelphia, Pa.

INEZ RUNDSTROM, Prof., Gustavus Adolphus Coll., St. Peter, Minn.

E. C. RUPP, M.S. (Denison). Asst. Prof., Denison Univ., Granville, Ohio.

PAULINE SHIRLEY, A.B. (Baylor). Teacher, High School, Wichita Falls, Texas.

A. K. SOLUM, A.B. (St. Olaf). Instr., St. Olaf Coll., Northfield, Minn.

RUTH L. SPAULDING, A.B. (Smith). Teacher, High School, Hartford, Conn.

J. H. STURDIVANT, A.B. (Texas). Instr., Univ. of Texas, Austin, Texas.

J. D. TAMARKIN, Ph.D. (Petrograd). Asst. Prof., Dartmouth Coll., Hanover, N. H.

ADELA M. THOM, A.B. (Kansas). Instr., High School, Okmulgee, Okla.

W. P. UDINSKI, Ph.D. (Illinois). University Junior Coll., San Antonio, Texas.

A. H. WAIT, A.M. (Wisconsin). Adj. Prof., Texas Tech. Coll., Lubbock, Texas.

A. D. WASHBURN, A.B. (Harvard). Head of Upper School and Head of Dept. of Math. and Sci., The Rivers School, Brookline, Mass.

E. D. WELLS, A.M. (Minnesota). Instr., Coll. of Eng., Univ. of Minn., Minneapolis, Minn.

ANNA M. WHITNEY, A.B. (Minnesota). Head of Dept., High School, Yakima, Wash.

ELLEN E. WILEY, A.B. (St. Lawrence). Asst. Prof., Middlebury Coll., Middlebury, Vt.

J. E. WILLIS, JR. Astronomer, U. S. Naval Observatory, Washington, D. C.

The resignation of Professor W. B. FORD as editor-in-chief of the MONTHLY, after four years' service, was accepted with regret and with an expression of deep appreciation of the high quality of scholarship and editorial ability shown in his administration. The Trustees appointed Professor W. H. BUSSEY of the

University of Minnesota as editor-in-chief, beginning with the volume for 1927 and feel very fortunate in securing Professor Bussey for this side of the Association's work.

The Trustees accepted invitations to hold the summer meetings of 1928 and 1929 at Amherst College and the University of Colorado, respectively. The same action was taken by the Council of the Society.

The resignation of Professor R. D. CARMICHAEL as chairman of the Committee on Standard Departments of Mathematics in Colleges was accepted and Professor TOMLINSON FORT was elected chairman, the president to fill the committee vacancy.

In recognition of the effort of the officers of the Louisiana-Mississippi Section to arouse the interest of teachers of mathematics, the president and secretary were authorized to send a letter to Professor S. T. Sanders, the chairman of the section, urging the cooperation of the high schools, normal schools and colleges of the two states, suggesting that they invite all secondary teachers of mathematics to attend the section meetings, that they devote some part of each program to questions of interest and concern to the secondary teachers, that they urge all these to subscribe to the *Mathematics Teacher*, that they urge all the collegiate teachers to join the Mathematical Association and invite any secondary teachers who wish to keep in touch with the collegiate field also to join the Association, the initiation fees of such teachers being waived provided they are already subscribers to the *Mathematics Teacher*, that they urge each school or institution to contribute toward the expenses of the teachers of mathematics in attending the meetings of the section, or at least to pay the expenses of one official delegate, the Trustees confidently believing that such expenditure of funds is sure to increase the worth and effectiveness of the teachers and thus to become a paying investment.

Later announcements will be made on business in progress concerning the Chace publication of the Ahmes papyrus, a proposed re-printing of the Report of the National Committee on Mathematical Requirements, and a proposed bureau of information as to vacancies in colleges.

W. D. CAIRNS, *Secretary-Treasurer*

THIRD ANNUAL MEETING OF NEBRASKA SECTION

The third annual meeting of the Nebraska Section of the Mathematical Association of America was held at Cotner University, Bethany, Nebraska, on April 30, 1926. in joint session with the Mathematics Section of the Nebraska Academy of Science.

The following papers were presented:

(1) "Definitions," by Professor R. M. MCDILL, Hastings College.

(2) "Some summable trigonometric series," by Mr. G. D. NICHOLS, University of Nebraska (by invitation).

(3) "Note on fundamental units in a pure cubic field," by Professor T. A. PIERCE, University of Nebraska.

(4) "Definitions of parallel curves," by Professor M. G. GABA, University of Nebraska.

1. In his paper on definitions, Mr. McDill made the plea that in mathematics definitions be short enough and be specific enough to be held in the mind by the pupil, that they be intelligible, even if it is impossible to make them both intelligible and free from all other criticism, that they stay in their place which is to furnish words to describe realities, though of course these realities may be thought realities which are idealized from physical realities, and that they do not insinuate principles. If we must assume certain principles as so self-evident as to seem fundamental, let us be honest and bold in acknowledging them as assumptions and not try to cover them under the cloak of definitions.

2. Using the Cesàro definition for the "sum," σ, of a non-convergent series, the following theorem was established:

$$\sum_{r=1}^{r=\infty} \frac{\phi(r)}{\psi(r)} \sin [c + (r-1)d]x \text{ and } \sum_{r=1}^{r=\infty} \frac{\phi(r)}{\psi(r)} \cos [c + (r-1)d]x$$

are k-ply indeterminate, where c and d are real positive, rational constants, and where

$$\phi(r) = a_s r^{\alpha_s} + a_{s-1} r^{\alpha_{s-1}} + \cdots + a_0,$$
$$\psi(r) = b_t r^{\alpha_t} + b_{t-1} r^{\alpha_{t-1}} + \cdots + b_0,$$

a_i, b_i, α_i, being any real, rational constants, and the α's being such that if they are arranged in descending order of magnitude $k-1 \leqq (\alpha_s - \alpha_t) < k$; $\psi(r) = 0$ having no positive integral roots; and the angle being restricted so that $dx \neq 2n\pi$.

A special case of the cosine series was treated, namely that in which the coefficient is a polynomial in r of the second degree. Thus, for $P(r) = r^2 + 3r$,

$$2^3 \sin^3 \frac{x}{2} \cdot \sigma = \lim_{n=\infty} \left\{ [\Delta^2 P(0) - 2\Delta P(0)] \sin \frac{x}{2} + \sum_{r=0}^{r=n-3} \frac{(n-r)(n-r+1)(n-r+2)}{(n+1)(n+2)(n+3)} \right.$$
$$\Delta^3 P(r) \sin \frac{2r+3}{2} x + \frac{60[3P(n) - 3P(n-1) + P(n-2)]}{(n+1)(n+2)(n+3)} \sin \frac{2n-1}{2} x$$
$$\left. - \frac{24[3P(n) - P(n-1)]}{(n+1)(n+2)(n+3)} \sin \frac{2n+1}{2} x + \frac{P(n)}{(n+1)(n+2)(n+3)} \sin \frac{2n+3}{2} x \right\}$$
$$= [\Delta^2 P(0) - 2\Delta P(0)] \sin \frac{x}{2} - \sigma \sin \frac{x}{2} \sigma = -\frac{3}{4} \csc^2 \frac{x}{2}.$$

Similar results would hold for polynomials of higher degree, but the actual process of obtaining the "sum" becomes more difficult the higher the degree of the polynomial.

3. In his note Mr. Pierce gave the results of some expansions by Jacobi's continued fraction algorithm. The numbers constituting the minimal basis of a pure cubic field were expanded, and fundamental units obtained in a large number of special fields.

4. Mr. Gaba based the definition of parallel curves on their relation to a common evolute and thus avoided a chance for ambiguity and misunderstanding.

At the business meeting the report of the nominating committee was adopted to the effect that the officers for the coming year are to be:

W. C. BRENKE of the University of Nebraska, Chairman; ELLEN H. FRANKISH of Omaha North High School, Omaha, Secretary-Treasurer; J. M. HOWIE of Nebraska Wesleyan University, Member of the Executive Committee.

EMMA E. HANTHORN, *Secretary-Treasurer.*

ON THE MECHANICAL HANDLING OF STATISTICS[1]

BY VICTOR JOHNS, 50 Broad Street, New York, N.Y.

The art of mechanical accounting—an accounting procedure that comprises the use of electric machines and perforated cards,—the application of which is now world-wide, germinated from the requirements of the United States Census Bureau. At the close of the compilation of the tenth census in 1880 the attention of Dr. Herman Hollerith, an engineer, who, as special agent of the bureau had won early recognition as an able and accomplished statistician, was drawn to the need of mechanical aid for census tabulation. For this purpose Dr. Hollerith developed a system of recording the descriptive data of each individual, or unit of inquiry, by punching holes in strips of paper, and later in cards. These perforations were adapted to control the

[1] The present article, which was prepared at the invitation of the editor, may be regarded as a sequel to the paper by Professor L. L. Locke, entitled, "The History of Modern Calculating Machines, an American Contribution," (1924, 422–429). The machines here described are essentially concerned with the handling of mathematical statistics, and are now extensively used throughout the world. Inasmuch as American colleges and universities, generally speaking, are developing courses in the scientific handling of statistics, and in many cases are installing the mechanical equipment necessary for such work, it is hoped that the present article may prove of interest and value to readers of the MONTHLY. Mr. Johns is an official of the International Business Machines Corp., of New York.—EDITOR.

tabulating machine—an electrically actuated mechanism in the form of a counting or adding device. Single classifications of data, or classifications in desired groups, were obtainable. The next use made of this tabulator, and of the supplementary machines devised for its application, was in the tabulation of mortality statistics in the city of Baltimore; it was also employed by the Bureau of Vital Statistics of New Jersey and by the Board of Health of New York City.

At the beginning of the organization of the eleventh United States census in 1890 a commission of three experienced statisticians was appointed by the Superintendent of Census to make a practical test of all tabulating systems available for use in the United States Census Bureau. The report of this commission showed that three methods were investigated and that the punched card tabulating machine method not only was found to be far more rapid than any other, but that in accuracy it was far ahead of the others. As a result, Dr. Hollerith's system of tabulation was selected for compiling the returns of the eleventh U. S. Census. The success attained attracted widespread attention, not only in the United States but also in foreign countries. About this time the same method was used in the compilation of the Austria-Hungarian census.

The cards used at the time were perforated by a machine known as the keyboard punch, a device that operated somewhat like a pantograph. The machine which counted or tallied the items punched in the cards was known as the tabulator. It resembled a small upright piano and consisted of two parts: (1) the press, or circuit-closing device, which was located in a position corresponding to the right-hand end of the keyboard; (2) the dials, or counters, which were located in a position corresponding to the upper face where the

FIG. 1
FIRST HOLLERITH
KEYBOARD PUNCH

FIG. 2
ORIGINAL MODEL OF THE POPULATION
TYPE HOLLERITH ELECTRIC
TABULATOR

music rack is set. A sorting box stood as a separate unit to the right of the machine. The bed plate of the press was formed by a series of mercury-filled holes, or cups, agreeing in number and arrangement with the holes that might be made on the cards. Above this was a reciprocating box provided with a number of contact points which corresponded in position with the center of the mercury cups, so that when a card was placed in the press, and the handle brought down, these points formed circuits in positions conforming to the holes in the punched records; the results were registered on the dials.

If, while certain facts were being tabulated, it was desired to sort or arrange the cards according to any data—as, for example, nationality—a sorting box was employed. This device was suitably divided into twenty-four compartments, each of which was closed by a lid held against the tension of a spring catch which formed the armature of a suitable magnet. These magnets were connected with the binding posts of the press, according to the data by which the cards were to be sorted. When a card was put in the press the armature corresponding with the given record was attracted, thus releasing the corresponding lid which remained open until the card was deposited in that division and the lid again closed by hand.

The use of electric tabulating machines for compiling population statistics presented the need of other mechanical devices for compiling agricultural, manufacturing and similar classes of data that dealt with quantities and amounts. Accordingly, a so-called integrating machine was developed. This device added digits from one to nine in each column, the accumulations being determined by the location of the holes in the card. This device was the predecessor of the modern electric tabulating machine now used extensively for railroad accounting, sales analysis, shop cost accounting, and similar purposes. Since tabulating machines can be used wherever figure-facts are called upon to assist in directing the operations of a business, the principle of punched hole accounting is represented in practically every field of human activity.

During the process of tabulating the eleventh census (1890) the attention of Dr. Hollerith was called to the need of some machine or device to aid in auditing railroad freight accounts and in calculating commodities statistics. Experiments were made with Hollerith tabulators at the office of the auditor of freight accounts of the New York Central Railroad. The test proved that the work could be done to advantage by this method. In the application of this system to railroad and commercial accounting, it was found desirable also to make radical changes in the apparatus for punching the cards, as well as in the apparatus by means of which the amounts were recorded.

One of the first units of the Hollerith system to be changed was the punching machine. In the pantograph type of punch, the fields, or groups of characters, on the cards were arranged in a sequence proceeding from the upper left-hand corner of the cards across the top and then from right to left across the bottom of the cards. In the work of punching cards for agricultural and railroad statistics, etc., it was found to be more convenient to have the card arranged so that the fields or classes of information would be in vertical columns—to facilitate the punching and reading of the cards as well as the tabulating. For this purpose the key punch was developed. While various improvements have been made from time to time on this device (illustrated by Figure 3) it is substantially the same in principle today as when devised. In 1923 an electrically actuated key punch was placed on the market. This machine requires a very light touch on the part of the operator and in many instances it has increased the production of card punching from ten to twenty percent.

FIG. 3
KEY PUNCH

Another recent development is the electric duplicating key punch, which can be used in the same manner and for the same purposes as those mentioned above, or it can be used to duplicate a portion, or all, of the perforations in a master card. When any information common to a number of cards is

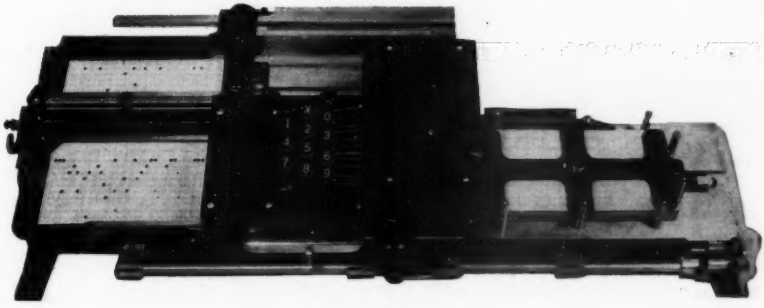

FIG. 4
ELECTRIC KEY PUNCH WITH DUPLICATING ATTACHMENT

to be duplicated, a master card is first punched and inserted in the rear carrier of the machine. The cards on which the data are to be duplicated are

then automatically fed from the card magazine to the punching device; the information on the master card is automatically reproduced on each of the desired number of cards. When the carriage reaches a point at which the individual data are to be manually punched, the machine stops and these data are recorded in the regular manner by the operator.

Another perforating device is the gang punch, which is used for recording those facts that are common to a number of cards, such as the month, day, year, etc. It is equipped with a number of movable punches which can be changed easily and set for any desired combination. The object of this machine is to provide a short cut in transcribing data. A number of cards—from ten to twelve—can be punched at one operation when the recorded data are similar for all the cards.

In order to check the accuracy of the punching, a so-called verification key punch was devised. As soon as the cards have been perforated the work is verified by merely inserting the punched cards, one at a time, in this device in the same manner as placing a fresh card in the key punch. The comparison is made by the operator pressing the keys in the same sequence as that followed when the data were transcribed from the original records. If the punching is correct the carriage and card will move forward to the next position. If the punching is incorrect, or if the wrong key of the verification key punch is depressed, the carriage will remain stationary.

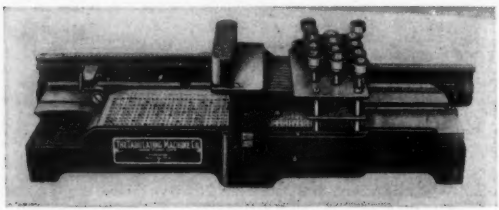

FIG. 5
VERIFICATION KEY PUNCH

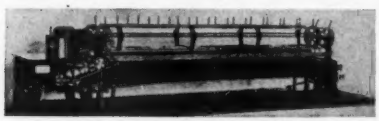

FIG. 6
ORIGINAL MODEL OF THE HOLLERITH ELECTRIC SORTING MACHINE

The unit which has made the operation of electric tabulating machines both practical and successful is the commercial tabulating card. While the number of columns on the cards has been changed to increase the recording capacity, the size of the cards, however, remains the same; which indicates on what a solid foundation of reason Dr. Hollerith's early work was based.

The development of the art of punched hole accounting continued; in 1902 the population and vital statistics of the United States were sorted and tabulated on machines having an automatic feed.

The sorting machine was redesigned, as shown by Figure 6, becoming fully automatic. Its next development was to place it in a vertical instead of horizontal position (Figure 7) as a means of conserving floor space. A stack of 400 or 500 cards is placed in the hopper at the top and automatically sent one by one into the twelve pockets below; the pocket into which a certain card falls depends upon the position of the hole in the column being sorted. Recently there was put on the market a new model (Figure 8), which operates almost silently and sorts cards at the rate of 400 per minute. Where conservation of floor space is not a matter of importance this machine is preferable from the operator's standpoint inasmuch as the pockets are all equidistant from the floor, which position brings all the sorted cards within easy reach.

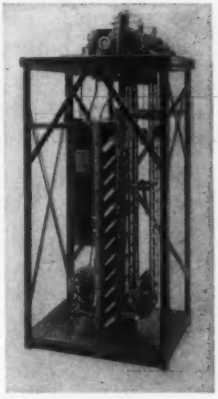

FIG. 7
ELECTRICAL VERTICAL SORTING MACHINE

At the time the sorter was redesigned a change was also made in the electric tabulating machine. The new principle involved—and still embodied in these machines—was this: As the cards drop from the hopper down through the

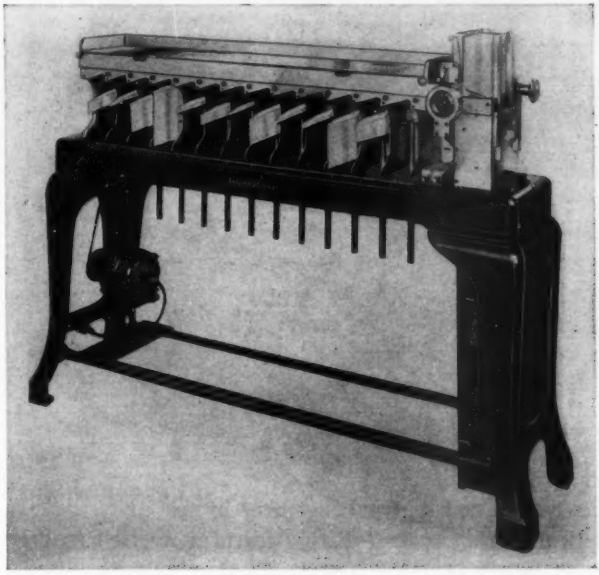

FIG. 8
RECENT FORM OF SORTING MACHINE

feed mechanism they pass between a row of small wire brushes and a row of fixed metal contacts corresponding in position to the columns of the cards, so that the punched holes, as the cards pass between the brushes and the metal contacts, cause electrical connections to be made, thus energizing magnets which in turn move the adding wheels. In this manner the amounts and quantities punched in tabulating cards are automatically totaled. Since as many as five different classifications of data can be added simultaneously on an electric tabulating machine at the rate of 150 amounts per minute for each classification—in other words, since an electric tabulating machine is capable of yielding *750 additions per minute*,—it is obvious that the power of this machine to handle great masses of numerical data is phenomenal.

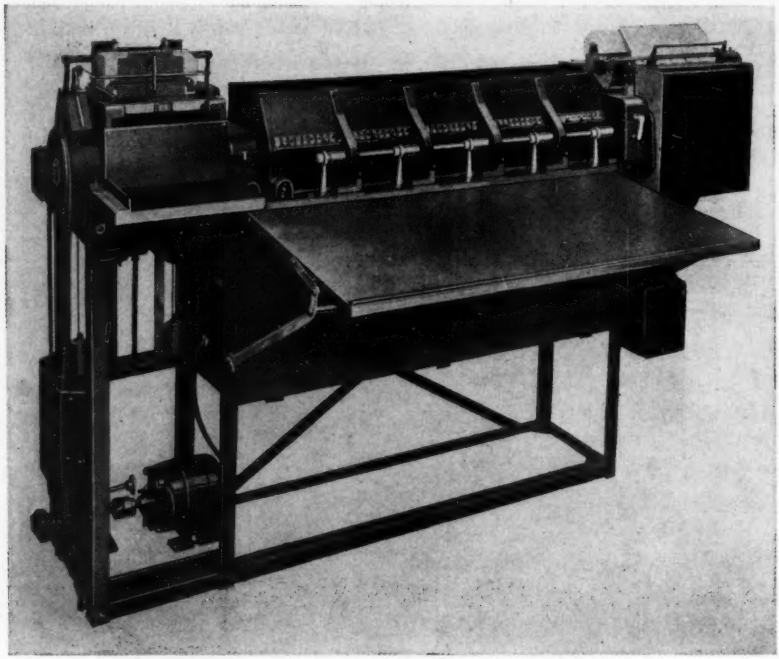

FIG. 9
ELECTRIC TOTAL PRINTING AND LISTING TABULATING MACHINE

There has been a constant improvement in this unit, the automatic control and the total-printing and listing features referred to below probably being the most important.

The automatically controlled tabulator was devised to eliminate the use of stop cards. Prior to 1922 it was necessary, when feeding various groups of

cards into the tabulating machine, to insert at the end of each group a stop card which, after the last card of a group had been registered, would stop the machine and the total then be recorded. The automatic control now in use does away with this extra card by automatically stopping the tabulating machine at the end of each group.

The introduction of the listing and total-printing feature was of major importance. With this device it is unnecessary for the operator to copy the figures shown on the counters, as the machine automatically prints on a sheet of paper the figures thus registered. It can be made to list simultaneously, in itemized detail and at the rate of 75 cards per minute, as many as seven different groups of data while accumulating the totals of five; when an itemized list is not desired, it accumulates and prints the totals of from one to five different groups at the rate of 150 cards per minute.

There is also the digit or card-counting tabulating machine which performs the functions of both the sorter and tabulator. This device is principally used in the compilation of employees and vital statistics. It is equipped with fourteen counters, twelve supplying individual totals, and two giving sub- or accumulated totals. It handles cards at the rate of 250 per minute.

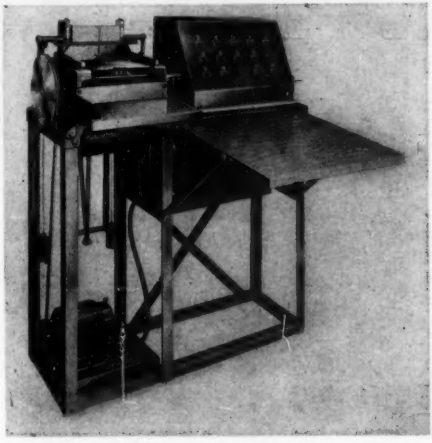

FIG. 10
ELECTRIC CARD COUNTING TABULATING MACHINE

The uses to which these mechanical accounting machines can be applied are unlimited: industry employs them for the maintenance of its records, for making production, cost, sales and many other analyses; insurance companies use them in the various phases of their statistical work; the United States and foreign governments use them for compiling agriculture and population censuses and for preparing multitudinous reports.

This equipment is also to be found in many of the great American colleges where it serves both practical and educational purposes: at the executive end it is used to prepare payrolls, to analyze direct labor charges and accounts payable, and to compile school and special study statements; at the educational end it is used for demonstration purposes in accounting courses. The University of Michigan, the LaSalle Extension, Johns Hopkins, Cornell and Columbia Universities are numbered among the users.

Electric tabulating machines, in fact, are used to record, aggregate, tabulate and print in report form all data capable of denotation by figures. They have brought to every angle of accounting work not only the facilities for saving time, labor, and money, but an absolutely new range of possibilities as well.

THE GREAT TREASURE HOUSE OF CHINESE AND EUROPEAN MATHEMATICS

By Père LOUIS VANHÉE, S. J., Brussels

The recent appearance in this MONTHLY of several interesting and valuable articles upon early Chinese mathematics suggests the desirability of calling attention to a notable source book that appeared in China in 1889 but which seems to be little recognized in the west. It is known as *The Great Treasure House of Chinese and European Mathematics* and was edited by Ch'en Wei-ki. It consists of a selection from some of the best mathematical treatises published in China, both native and European, and is made up of a hundred parts, some of these being classical treatises and others being mere compilations. The following list with comments, affords an idea of the scope of this monumental compendium:

1. (Part 1). The *lo-shu* and *ho-t'u*; the *Chou-peï Suan-King*;[1] Measures. It is a general custom in China to have a mathematical treatise begin with the *lo-shu* and *ho-t'u*, figures of fabled origin,[2] the former being the world's first known magic square and the latter being used for divination. The *Chou-peï Suan-king* is well known to the western world through Biot's translation[3] and has only a sentimental value. It pretends to be a dialogue between Chóu-Kung and Shang Kao, shortly before 1100 B. C., but this is probably a literary fiction devised for the purpose of giving the impression of an advanced state of mathematical knowledge in that period. The style is obscure and the work is merely a set of vague statements about numbers derived from the circle and the square.[4] The Pythagorean theorem is, with their usual obstinacy in such matters, claimed by the Chinese, but the impartial historian must exert a great deal of charity to sanction this appeal to national pride.

2. (Parts 2–17). Euclid's *Elements*, the first six books being the translation made by the celebrated mandarins Sü Kuang-ch'i[5] and Li Chi-ts'ao (died in 1631), under the direction of the far more celebrated Matteo Ricci

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[1] Smith, *History*, I, 28, 29; This MONTHLY (1925, 499–504).

[2] Smith, I, 29.

[3] *Journal Asiatique* (1841), p. 595.

[4] See the extracts in Smith, I, 31.

[5] Known after his adoption of Christianity as Paul Sü (Hsü, Siu) (1562–1634). Smith, I, 304.

(1552–1610). This remarkable Jesuit scholar, intimately associated with the savants of the Empire, sought to give to the Chinese a sound mathematical basis upon which to build their own science and philosophy. Although he died before the influence of his work was fully appreciated the world has not failed to pay to his memory the tribute that is so justly due. Himself a pupil of the great Clavius (1537–1612), whose edition of Euclid appeared (1574) three years before Ricci left for China, it was this text that he followed in making the translation. He first explained to Sü Kuang-ch'i the meaning of each proposition; he then dictated the translation; the minister then took this to his home and rewrote it in the polished style of the Chinese scholar; and finally Ricci revised the text for publication. This intimate association between two great scholars, one from the east and the other from the west, gave to the Chinese civilization this great classic, known in their tongue as the *Ki-ho-yüan-pen*.[1] The style is remarkable, reading as pure Chinese and not as a translation from the Greek or Latin, and thus the work is in marked contrast to most oriental translations from the European languages, hastily made as they are by immature writers and immediately thrown on the market merely with the idea of ready sale. The remaining books of the *Elements* were translated from the English by Wylie (1815–1881) with the assistance of Li Shan-lan (1814–1884).

3. (Parts 18–22). "Calculation with the pen." This relates to ordinary written computation as distinct from that on the abacus. It contains but little theory, being a collection of all sorts of possible as well as imaginary applications.

4. (Parts 23–25). Proportion. A collection of problems from various sources.

5. (Part 26). The *Ts'ié-shuai* method, an extension of the method of proportion. The expression means literally "borrowing of terms" and is used to indicate problems in which the data are not sufficient to enable the computer to write the proportions at once, quite as in the case of an indeterminate equation.

6. (Part 27). Excess and deficiency. This term, familiar in medieval Europe as a synonym of *Regula falsi positionis* or "Rule of False Position," is an old one in China. It is usually employed there with respect to problems like the following:

"A certain sum of money is divided among a certain number of men. If each takes 7 taels, 4 taels remain; but if each should demand 9 taels, there would be 12 taels too few. Required the number of men and the amount of money."

[1] The *Ki-ho* means "how much," "quantity," "magnitude," and is the most elegant way of translating our word "geometry." In spite of the influx of new terms, the native form is still used, the syllable *hsiao* (science) being added, giving the familiar form *Ki-ho-hsiao*. The syllables *yüan-pen* signify "original volume," or "first text," *yüan* meaning "origin" and "*pen*" meaning "volume."

"A crowd of people wish to be taken across a stream. If each boat carries 13, there will be 12 people left; but if each boat carries 18, there will be one boat too many. Required the number of boats and the number of people."

7. (Part 28). Equations, *Fang-ch'eng*. The term *Fang* means "to compare," and *ch'eng* means "system" or "form." The title refers to the solution of numerical linear equations with two or more unknowns. No theory is given, showing that the Chinese had merely borrowed a mechanical method from foreign sources.[1]

8. (Parts 28–30). *Kou-ku* (Right triangle). This is the title of the last section of the celebrated *K'iu-ch'ang Suan-shu* (Arithmetic in Nine Sections), one of the oldest of the Chinese classics.[2] It is a subject frequently included in the mathematical works of China, where problems involving the Pythagorean theorem are a favorite pastime. It involves nothing of any difficulty or novelty.

9. (Part 31). Plane trigonometry, evidently merely a translation from European sources.

10. (Part 32). Measure of heights and distances. A very simple application of proportion, with the first principles of trigonometry.

11. (Part 33, 34). The circle. The quadrature problem seems to have interested Chinese scholars from very early times. Many names of such writers are found in the Chinese encyclopedias. It is the writer's belief that all the circle theory contained in these works was derived from western sources. Indeed, Greek and other foreign writers are frequently mentioned. It is unfortunate that the Chinese reader is misled by such titles as "Method of Jartoux," "Method of Yu," and "Method of Li." In the case of Jartoux, (1668–1720), the well-known French Jesuit missionary, the error is made the worse by the use of his Chinese name, Tu, and by the impression which is given that Li (who was merely an assistant of Wylie's) and Yu are to be ranked in the same class with a man whose nine series for quadratures are so highly appreciated in the east.

12. (Parts 35, 36). Surfaces. A series of applied problems requiring only a small amount of elementary mathematics.

13. (Parts 37, 38). Solids. Similar in treatment to **12.**

14. (Part 39). Spherical trigonometry. China received her knowledge of this subject from two sources; (1) the Arabs in the 13th century, and (2) the Jesuits in the 17th century. The first of these sources did not appreciably contribute to her mathematics; the second gave her a set of practical formulas

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[1] This, however, may not be a correct inference, since it was not an uncommon thing even for European scholars in the early Renaissance period to conceal the method of solution of certain types of problems.

[2] Smith, I, 31.

the basis of which she only vaguely comprehended and the potency of which she failed to recognize by making any advance in the theory.

15. (Parts 40–50). "The algebra of the celestial element," a name derived from the fact that in China the unknown was represented by the character *t'ien* (heaven). This algebra apparently of Hindu or Arabic origin, was adapted (with numerous linguistic curiosities) to the Chinese language and appears in the works of such prominent writers as Li Yeh (1178–1265) and Yang Hui (*c.* 1260). The compiler embodied in his work everything that the Chinese had attempted in this line, including 70 of the 170 problems in Yeh's *Ts'é yuen hai king* (*Sea Mirror of the Circle Measurement*) (1249);[1] the treatment of progressions by Chu Shih-Kié (Chóu Che-Kié, *c.* 1300) and Li Shan-lan (1815–1884); square root according to Li Juan and Chiu Pai-Ki (1819–1869); and finally a conjectured explanation by Hao of the ancient method of finding roots.

16. (Parts 51, 52). Quadrilateral algebra of Chu Shih-Kié (Chóu Che-Kié) (*c.* 1300) so called because it employs for the unknowns four Chinese characters, *t'ien*, *ti*, *jen*, and *wu* (*heaven*, *earth*, *man*, and *thing*). By a special disposition of these characters it is able to indicate powers without resorting to exponents. Such a use of four letters may possibly be due to the Hindu or Arab influence.[2]

17. (Part 53). The *Chui* method. The term *Chui* is obscure and its meaning has been the subject of much dispute. It seems to have been first used in a work by Tsu Ch'ung-chih (430–501)[3] the *Chui-Shu* (*Book of Chui*) and probably referred to the treatment of the calendar. In later times, however, it has been employed by both Chinese and Japanese writers to refer to the analytic treatment of the quadrature of the circle, and is so used in this collection. It is owing to this change in meaning that certain writers have attributed to Tsu-Ch'ung-chih the analytic method of finding the value of π.

18. (Parts 54, 55). Elementary algebra. An adaptation of the *Ts'ie-ken-fang* of the Jesuit scholars of the 17th century.

19. (Parts 56, 57). Logarithms. This feature was introduced into China by the Polish Jesuit Smogolenski[4] (1611–1656) who, knowing that the Chinese of that period were not prepared for the theory, gave only the tables[5] and the mechanical rules for using them. In the 19th century the influence of the west led to a demand for the underlying principles, and these were adapted from English works, but with an absurd claim for originality for three Chinese writers, Ku, Li, and Tsou.

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[1] Smith, I, 270.

[2] Compare the Hindu use of various colors for the unknown.

[3] Smith, I, 143.

[4] In Chinese, Mu Ni-ko. This is derived from the original Polish form—Nicolas Smogulecki, *Ni-ko* being from *Nico*las, and *Mu* being from the syllable *Smo*.

[5] Described in his *T'ien-pu Chen-yüan*.

20. (Parts 58–79). *T'ai-Shu*, a name given to advanced algebra. This includes the more elementary parts of modern algebra and contains numerous applications to geometry, conics, and other branches.

21-23. (Parts 80–96). Differentiation, integration, and mechanics. Translations from English texts, including that of the American scholar, Elias Loomis on conics, and the calculus.

24. (Part 97). The slide rule.

25. (Part 98). Logarithmic tables. The Jesuits had, as stated under **19**, introduced logarithms in the 17th century. So remarkable did this invention seem that the emperor, K'ang-Hsi, had a small table prepared for his own use. The compilation known as the *Shu-li Tsing-yün*, a part of the *Lü-li Yüan-yüan* printed at Peking in 1713, reproduced the Vlacq tables of 1628.[1]

26, 27. (Parts 99, 100). Trigonometric functions, natural and logarithmic. Reproductions from European sources.

The compilation, prepared for the purpose of stimulating mathematical work at a time when European influence was at its maximum and indicative of the radical changes that have since taken place, has great historical interest. In its treatment of the subject there is always apparent a lack of the judicial mind and there are constant indications of a characteristic spirit. Yé, for example, recognizes the value of the work of Diophantus, but he asserts that the ancient Chinese algebra was the source rather than the outcome of the works of the Hindus, Greeks, and Arabs, and that foreign writers improved upon the earlier works of the far east. As a kind of source book for the history of Chinese mathematics, however, the work has a value which justifies making it known to western scholars.[2]

THE KINETICS OF LEARNING

By H. J. ETTLINGER, University of Texas

The laws of certain specified types of the learning process, which may be roughly represented by learning to operate a typewriter, may be embodied in a first order linear differential equation (or difference equation) with constant coefficients. From this relation follows a simple set of equations which express chiefly by means of exponential functions (or binomial expressions) the quanti-

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[1] Smith, I, 436; II, 524.

[2] Professor D. E. Smith, in translating this article for the MONTHLY from Père Vanhée's manuscript, materially revised and shortened it. The original article has since appeared in *Archivio di Storia della Scienza* for May, 1926.

tative aspects of the learning process and exhibit it as a form of "growth." It is not unlikely that equations of this kind may prove useful in analyzing personnel in factory and offices, as well as in predicting future performances involving the above type of learning process from a limited amount of data obtained by observation.

We shall make the following assumptions:

1. The *attainment*, Y, may be measured in terms of the number of successful acts per unit of time (practice period).

2. There exists a *limit of attainment*, L, measured in terms of the number of successful acts per unit period of time.

3. The difference $y=L-Y$, limit of attainment minus attainment is the *margin of attainment.*

4. The amount of practice acts, X is a continuous variable.

5. When $X=0$, the attainment, Y, is zero.

The *Law of Learning* may be stated as follows:

The relative rate of decrease of the margin of attainment with respect to the amount of practice is constant.

If y' is the derivative of y with respect to X, we may write the Law of Learning as a linear differential equation of the first order,

$$(y'/y) = -k, \tag{1}$$

where k is a constant. Integrating (1) we obtain

$$\log y = -kX + C.$$

When $X=0$, $y=L$, hence

$$y = Le^{-kX}.$$

Since $y=L-Y$, we have for the equation connecting Y and X,

$$L - Y = Le^{-kX} \tag{2}$$

or

$$Y = L(1 - e^{-kX}) \; ; \; X \geqq 0.$$

The graph of the equation (2) is shown in figure 1. It is the typical *growth* curve or compound interest law. We note that when $X=0$, $Y=0$, and as X increases without limit, Y approaches L as a limit.

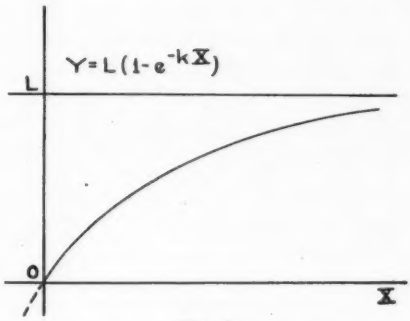

FIG. 1

Now from (2) we obtain

$$\frac{L}{Y} = \frac{1}{1 - e^{-kX}} = 1 + \frac{e^{-kX}}{1 - e^{-kX}} = 1 + \frac{1}{kX} - \frac{e^{kX_1}}{2 + kXe^{kX_1}}$$

by applying the Law of the Mean[1] to the term $e^{-kX}/(1-e^{-kX})$, where X_1 is an intermediate value of X, $0<X_1<X$. The equation

$$\frac{L}{Y} = 1 + \frac{1}{kX} \text{ or } Y + kXY = kLX,\ X > 0, \tag{3}$$

may be regarded as a first approximation to (2). The graph of (3) shown in figure 2 is a *hyperbola*, with a vertical asymptote at $X=-1/k$ and a hori-

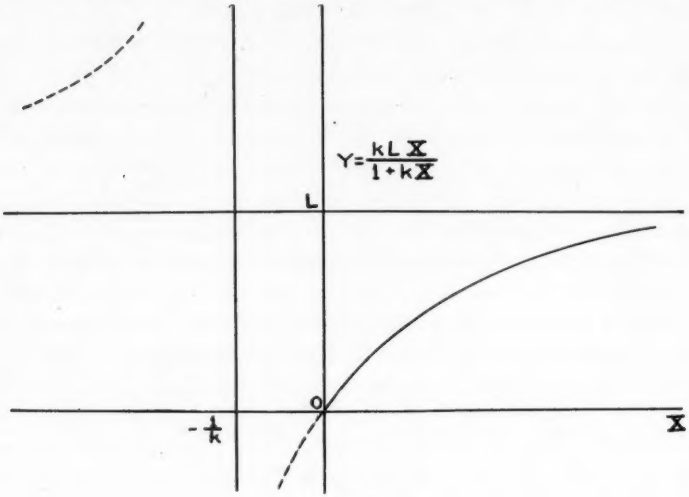

FIG. 2

zontal asymptote at $Y=L$. Furthermore it passes through the origin since $Y=0$ when $X=0$.

This hyperbolic curve was obtained empirically by the psychologist Thurstone[2] for a learning curve from an extended series of typewriter learning data. As a practical method for determining (2) empirically, one may find the value of k and L in equation (3) by the use of the equation preceding (3). Plotting $1/Y$ against $1/X$, we obtain the vertical intercept as $1/L$ and the slope of the straight line as $1/kL$. From these, k and L may be computed and substituted into (2).

If we set $t=1/Y$, $t_L=1/L$, where t is the attainment measured in number of units of time required to perform unit successful act, we have

$$t = t_L(1 - e^{-kX})^{-1}. \tag{4}$$

[1] This result may also be obtained by the well known method of integration by parts as applied to approximations. See, for example, Porter and Ettlinger, *Elementary Calculus*, The Century Company, 1925, pp. 209–210.

[2] Thurstone, The Learning Curve Equation, *Psychological Monographs*, Studies from the Psychological Laboratories of the University of Chicago, vol. xxvi, no. 3, whole no. 114 (1919) pp. 1–51.

If we interpret t as the time rate of change of time of practice T, with respect to the amount of practice, X, we have a second differential equation

$$\frac{dT}{dX} = \frac{t_L}{1 - e^{-kX}}$$

which may be integrated to yield

$$T = \frac{1}{kL} \log \frac{e^{kX} - 1}{e^{kP} - 1}, \tag{5}$$

where P is the value of X corresponding to $T = 0$.

Equation (5) may be called with Thurstone[1] the *time-amount* curve. We may solve this equation for e^{kX} and substitute in (4) and obtain

$$t = t_L + (t_P - t_L)e^{-kLT} \tag{6}$$

where t_P is the time corresponding to $X = P$. Equation (6) is Thurstone's *time-time curve*,[2] that is, the attainment measured in time units, in terms of time of practice.

The reciprocal of (6) gives the *speed-time* curve,

$$Y = \frac{LY_0}{Y_0 + (L - Y_0)e^{-kLT}} \tag{7}$$

represented in figure 3.

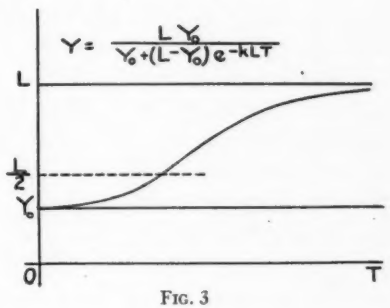

FIG. 3

By the methods of elementary calculus it may be shown that curve (7) has a point of inflection at $Y = L/2$. This result is more easily obtained from the inverse equation of (7),

$$T = \frac{1}{kL} \log \frac{Y(L - Y_0)}{Y_0(L - Y)}.$$

The slope of curve (7) is the rapidity with which the speed of attainment increases with time of practice. The point of inflection yields the value for which this rate is a maximum.

As pointed out above, a limited number of observations will permit the determination of L, the limit of attainment. The determination of this quan-

[1] *Loc cit.*, p. 21.

[2] *Loc cit.*, p. 21. Thurstone is unable to obtain an equation of this type in explicit form from his hyperbolic approximation. For a complete comparison between the present equations and those obtained by Thurstone, see my paper entitled "Representation of the learning process by means of a growth curve," read before the Psychology Seminar of the University of Texas, cf. *Journal of Experimental Psychology*, October, 1926.

tity in advance would give important information as to what may be expected of the future performance of the individual. This may prove useful to efficiency experts. It may also be of interest in another field, *viz.*, athletic coaching. The instructor is not interested in the present attainment of an individual, but in what may be expected at the height of the season after a large amount of practice.

The assumption has been made above that when $X=0$, $Y=0$. In many learning processes this condition is not fulfilled. We may have a certain initial attainment without any practice. In this case, when $X=0$, $Y=Y_1$. The latter case, however, reduces to the former by setting $\overline{Y}=Y-Y_1$.

Equation (1) may be turned into a Law of Forgetting by changing the sign of k and properly interpreting the quantities involved.

It may be pointed out that equation (1) should in reality be a difference equation with intervals of unity for the independent variable. Such an equation may be written $\Delta y_i = -ky_i$, or $y_{i+1} = y_i(1-k)$, $y_n = y_0(1-k)^n$. (8) But $y_0=L$, and $y_n=L-Y_n$, hence

$$Y_n = L[1-(1-k)^n], \tag{9}$$

where n is the number of practice periods.

Equation (9) is defined only for integral values of n, but in the main the properties of the curve, if we join these points by straight lines, will be very much like those of (2). For $n=0$, $Y_0=0$. If k is positive and less than unity, as n increases without limit, Y_n will approach L as its limit. For large values of n we have from the equation $(L/Y)=1/[1-(1-k)^n]$, an approximation similar to (3) of the type,

$$Y = kLn/(1+kn).$$

Similarly we may obtain other relations analogous to those obtained for the differential equation.

QUESTIONS AND DISCUSSIONS

EDITED BY TOMLINSON FORT, Hunter College, Park Ave. and 68th St., New York, N. Y.
AND BY H. E. BUCHANAN, Tulane University, New Orleans, La.

The department of Questions and Discussions in the Monthly is open to all forms of activity in collegiate mathematics, including the teaching of mathematics, except for specific problems, especially new problems, which are reserved for the separate department of Problems and Solutions.

DISCUSSIONS

I. A VECTOR PROOF OF THE THEOREM OF CORIOLIS

By E. L. REES, University of Kentucky

The following is a very neat vector proof of the theorem of Coriolis. We are to show that the absolute acceleration of a point in motion relative to a

moving body is made up of the following components: its relative acceleration; the acceleration of the point of the body with which it instantaneously coincides; and a component depending on the relative velocity of the point and the angular velocity of the body.

Let $\boldsymbol{p}$ be the position vector of any point P fixed in the body and $\boldsymbol{q}$ the vector from P to the moving point Q. The position vector of Q is then given by $\boldsymbol{r}=\boldsymbol{p}+\boldsymbol{q}$.

$\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are of course functions of the time. Differentiating twice and indicating time derivatives by dots we have $\ddot{\boldsymbol{r}}=\ddot{\boldsymbol{p}}+\ddot{\boldsymbol{q}}$.

To introduce the relative motion of Q we refer $\boldsymbol{q}$ to the mutually perpendicular unit vectors $\boldsymbol{i}'$, $\boldsymbol{j}'$, $\boldsymbol{k}'$ fixed in the body. A vector expressed in terms of $\boldsymbol{i}'$, $\boldsymbol{j}'$, $\boldsymbol{k}'$ we denote by a primed letter. Thus we may write

$$= \Sigma \boldsymbol{q} \cdot \boldsymbol{i}'\boldsymbol{i}',$$

in which $\boldsymbol{i}'$, $\boldsymbol{j}'$, $\boldsymbol{k}'$ of the coefficients are to be regarded as functions of the time. Differentiating this equation, we have

$$\dot{\boldsymbol{q}}' = \Sigma \dot{\boldsymbol{q}} \cdot \boldsymbol{i}'\boldsymbol{i}' + \Sigma \boldsymbol{q} \cdot \dot{\boldsymbol{i}}'\boldsymbol{i}' = \dot{\boldsymbol{q}} + \Sigma \boldsymbol{q} \cdot \boldsymbol{w} \times \boldsymbol{i}'\boldsymbol{i}'$$

$$= \dot{\boldsymbol{q}} + \Sigma \boldsymbol{q} \times \boldsymbol{w} \cdot \boldsymbol{i}'\boldsymbol{i}' = \dot{\boldsymbol{q}} + \boldsymbol{q} \times \boldsymbol{w}.$$

By a second application of this formula, we have for the second derivative

$$\ddot{\boldsymbol{q}}' = \ddot{\boldsymbol{q}} + 2\dot{\boldsymbol{q}} \times \boldsymbol{w} + \boldsymbol{q} \times \dot{\boldsymbol{w}} + (\boldsymbol{q} \times \boldsymbol{w}) \times \boldsymbol{w}.$$

Now let Q coincide with P so that $\boldsymbol{q}=0$ and $\dot{\boldsymbol{p}}'=\dot{\boldsymbol{q}}$. The last equation then becomes

$$\ddot{\boldsymbol{q}}' = \ddot{\boldsymbol{q}} + 2\dot{\boldsymbol{q}}' \times \boldsymbol{w}.$$

Solving for $\ddot{\boldsymbol{q}}$ and substituting in the equation above, we get

$$\ddot{\boldsymbol{r}} = \ddot{\boldsymbol{p}} + \ddot{\boldsymbol{q}}' + 2\boldsymbol{w} \times \dot{\boldsymbol{q}}',$$

which proves the theorem. The component represented by the last term is known as the acceleration of Coriolis.

II. Notes on Some Uses of the Line at Infinity and of Imaginaries

By A. D. Campbell. University of Arkansas

In these notes we shall show some interesting uses of the line at infinity (l_∞) and of imaginaries, first in the general discussion of simultaneous real quadratic equations in two unknowns and secondly in the study of real plane

cubic curves in Euclidean geometry. We worked out complete results in these two problems with a graduate student, Mr. Sam Byrd. We shall give two illustrations of our method.

We note that the discussion in algebra of two simultaneous real quadratic equations in two unknowns and of their common solutions corresponds to the discussion in geometry of two conics in the real affine plane projective geometry and of the points of intersection of these two conics. Moreover the two conics may be considered as fundamental conics of a pencil of conics. Therefore we may replace the algebraic problem of finding typical forms to which all such simultaneous quadratic equations can be reduced by the geometric problem of finding typical forms to which all pencils of conics in the real affine geometry can be reduced. We define a class of pencils of conics as the set of all pencils projectively equivalent to a given typical pencil. We note that the classes of pencils of conics in the real affine geometry can be derived as subclasses of the classes in the real general projective geometry.[1] We shall now use this last mentioned fact. The first illustration of our method will be the finding of all the classes of pencils of conics in the real affine geometry (together with their typical pencils) that are subclasses of the class in the real general projective geometry that has for a typical pencil:[2]

$$\lambda(x^2 + y^2) + \mu(x^2 + z^2) = 0, \tag{1}$$

x, y, z being homogeneous coordinates referred to a triangle of reference. (To show more clearly how our geometrical discussion solves also the algebraic problem, we shall put our results in the form of simultaneous quadratic equations rather than of pencils of conics.) Each pencil of this class consists of all the conics that intersect in two pairs of conjugate imaginary points, P_1, P_2 and P_3, P_4 that form the vertices of a complete quadrangle. The degenerate conics of the pencil are two conjugate imaginary line pairs P_1P_3, P_2P_4 and P_1P_4, P_2P_3 and one real line pair P_1P_2, P_3P_4 that form the pairs of opposite sides of this complete quadrangle.

We have the following cases arising from the different possible positions of l_∞ relative to the degenerate conics of a pencil in the above class: (*a*) l_∞ does not coincide with a side of the complete quadrangle P_1, P_2, P_3, P_4, nor pass through a vertex or a diagonal point, (*b*) l_∞ passes through the diagonal point $D(P_1P_4, P_2P_3)$, (*c*) l_∞ passes through D and the diagonal point $D'(P_1P_3, P_2P_4)$, (*d*) l_∞ is the side P_1P_2, (*e*) l_∞ passes through the diagonal point $D''(P_1P_2, P_3P_4)$, (*f*) l_∞ is the line DD''.

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[1] See Veblen and Young *Projective Geometry* vol. II, pp. 70, 71.

[2] See L. E. Dickson "On Families of Quadratic Forms in a General Field" in the *Quarterly Journal of Mathematics*, vol. 39, pp. 316–333.

Case (a) gives (if we choose the x- and y-axes suitably and take two of the degenerate conics of the pencil for fundamental conics and their equations as the typical simultaneous quadratic equations we are seeking)

$$xy = 0,\ \left(\frac{x}{a+i}+\frac{y}{b+i}-1\right)\left(\frac{x}{a-i}+\frac{y}{b-i}-1\right)=0, \tag{2}$$

where $i=\sqrt{-1}$, with seven other equivalent cases in which the denominators are as follows:[1]

$$(-a-i,-b-i,-a+i,-b+i),(a+i,-b+i,a-i,-b-i),$$
$$(-a+i,b+i,-a-i,b-i),(b+i,a+i,b-i,a-i),$$
$$(-b+i,-a+i,-b-i,-a-i),(-b+i,a+i,-b-i,$$
$$a-i),(b+i,-a+i,b-i,-a-i).$$

To get all the equivalent cases we first note that the pencil of conics corresponding in geometry to (2)[2] is equivalent to any pencil (2′) with the same denominators as (2) but referred to a different set of axes from those to which (2) is referred. Therefore to find all the pencils of type (2′) that are equivalent to (2), besides the cases where $a'=a$ and $b'=b$, we have merely to find the pencils (2′) that are equivalent to (2) and are also referred to the same axes as (2). To find the transformation that will send (2) into (2′) we note which degenerate conic of (2) goes into the degenerate conic $xy=0$ of (2′) and which goes into

$$\left(\frac{x}{a'+1}+\frac{y}{b'+1}-1\right)\left(\frac{x}{a'-i}+\frac{y}{b'-i}-1\right)=0.$$

Cases (b) through (f) give us respectively the following cases:

$$xy = 0,\ \left(\frac{x}{a+i}+\frac{y}{-a+i}-1\right)\left(\frac{x}{a-i}+\frac{y}{-a-i}-1\right)=0, \tag{3}$$

with one equivalent case in which the denominators are:

$$(-a+i,a+i,-a-i,a-i).$$

$$xy = 0,(ix+iy-1)(ix+y+1)=0. \tag{4}$$

$$y = 0,(y+ix-1)(y-ix-1)=0. \tag{5}$$

$$x(x-1)=0,[(1+a)x+iy-1][(1+a)x-iy-1]=0, \tag{6}$$

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[1] The transformation $x=-x'$, $y=-y'$ gives the first equivalent case. Similar transformations give the other cases.

[2] We shall call this pencil the pencil (2).

with equivalent cases with coefficients: $(1-a, i, 1-a, -i)$,

$$\left(\frac{1}{1+a}, i, \frac{1}{1+a}, -i\right), \left(\frac{a-1}{a}, i, \frac{a-1}{a}, -i\right), \left(\frac{a+1}{a}, i, \frac{a+1}{a}, -i\right),$$

$$x(x-1) = 0, \quad (y-i)(y+i) = 0. \tag{7}$$

As an illustration of how we applied the above method to the study of real plane cubic curves in the Euclidean geometry, we shall consider the class of cubics in the real ordinary projective geometry with a typical cubic:

$$y^2z = x^3, \tag{8}$$

referred to a triangle of reference. Each cubic of this class has a cusp P_1 with its tangent l_1, a point of inflection P_2 with its tangent l_2 and its harmonic polar[1] l_1. In the cubic (8) P_1 is (0, 0, 1), P_2 is (0, 1, 0), l_1 is $y=0$, l_2 is $z=0$. We have the following cases according to the different positions of l_∞ relative to the cubic: (*a*) l_∞ is l_1, (*b*) l_∞ is l_2, (*c*) l_∞ is the line l_3 $(P_1P_2,)$ (*d*) l_∞ passes through P_1 but is not l_1 or l_3, (*e*) l_∞ passes through P_2 but is not l_2 or l_3, (*f*) l_∞ passes through the point $P_3(l_1l_2)$ but is not l_1 or l_2, (*g*) l_∞ cuts the cubic in three real and distinct points but does not pass through P_1 or P_2 or P_3, (*h*) l_∞ touches the cubic but does not pass through P_1 or P_2 or P_3, (*i*) l_∞ cuts the cubic in one real and two conjugate imaginary points but does not pass through P_1 or P_2 or P_3.

The above cases (*a*) through (*f*) give us respectively the following typical cubics in the real affine geometry:[2]

$$y = x^3, y^2 = x^3, y^2x = 1, y^2x = (y+1)^3, y^2x = (x+1)^3, y^2(y+1) = x^3.$$

Cases (*g*), (*h*), (*i*) give us:[3]

$$y^2(x+y-1) = \alpha x^3, \alpha \neq 0. \tag{9}$$

According as α makes the cubic $y^3+y^2x-\alpha x^3=0$ (which gives the intersections of the curve with l_∞) have three real and distinct roots in y/x, or two equal roots, or one real and two conjugate imaginary roots, do we have cases (*g*), (*h*), (*i*) respectively. Just as in the study of simultaneous quadratic equations, so here we can readily prove that (9) is not equivalent to any (9′) with $\alpha' \neq \alpha$.

We now wish to arrive at the cubic curves in the Euclidean geometry. We take as x'-axis in a set of rectangular axes for (9) the original x-axis of (9) and as y'-axis a line making an angle $\pi/2-\omega$ with the old y-axis. Then we perform the two transformations:

$$x = x' - y' \cot \omega, \quad y = y' \csc \omega. \tag{10}$$

$$x' = ax'', y' = by'', ab \neq 0, \tag{11}$$

[1] See Hilton *Plane Algebraic Curves*, page 95.

[2] Compare Salmon *Higher Plane Curves*, Third Edition, Art. 209.

[3] The line l_3 will have an equation of the form $ax+by-1=0$, and the resulting cubic $y^2(ax+by-1) = x^3$ is reducible to (9).

with no case of $a=\pm 1$, $b=\pm 1$. Using (10) and (11) on (9) we get:

$$b \csc \omega y'' = (ax'' - b \cot \omega y'')^3, \tag{12}$$

where (12) is not equivalent to a similar cubic (12′) unless $a'=\pm a$, $b'=\pm b$, $\omega'=n\pi\pm\omega$.

RECENT PUBLICATIONS

EDITED BY W. B. CARVER, Cornell University, to whom books and communications should be sent.

REVIEWS

Espaces courbes. Critique de la relativité. By C. BURALI-FORTI AND T. BOGGIO. Torino, Sten Editrice, 1924. xxiv+255 pages.

This book by two known Italian mathematicians makes one feel sad. It is an example of how intolerance can mislead even powerful minds in a field where we would least expect it.

To explain the situation it will be necessary to make first a few general remarks. The two recognized methods in geometry, the synthetic method (of which Euclid's treatment of the so-called elementary geometry is an outstanding example) and the analytic, have each its advantages and its defects. The advantage of the synthetic method is that it deals directly with the objects of the study while the corresponding defect of the analytic method is that it introduces extraneous things like the axes of coordinates. On the other hand, the analytic method has the advantage that it uses the powerful instrument of formulas—it makes the formulas work for us. It is natural that as early as Leibniz attempts were made to devise a new method—a direct geometrical analysis—which would embody the advantages of the other two without having their defects. We cannot go into the most interesting history of this subject; it will suffice to remark that at present there is no lack of systems of direct geometrical analysis; the trouble is rather that there are too many of them. The number of different geometrical languages is immense and their use is often accompanied by manifestations of partisanship, or a sort of nationalism, lack of tolerance toward other systems, and pugnacity. The most militant of these systems originated in connection with an attempt at unification of notations; C. Burali-Forti, one of the authors of the book under review, is the moving spirit of this school.

It must be said, however, that the situation is by no means hopeless; no great importance can be attached to the use of one or another system of notations; this is largely a matter of taste and, even more, of habit; it may be mentioned parenthetically that the reviewer has been using in his work various of these systems and he finds that it is easy (and useful) to learn how to pass from one system of notations to another.

In the treatment of curved spaces which the advent of the so-called general relativity theory brought into such prominence, the importance of direct methods is even more pronounced than in ordinary geometry. The use of extraneous elements leads in this field to the introduction of distinctions which have nothing to do with the geometric objects and are due only to the system of representation used; for instance, we begin to see double; instead of one vector we see two, a covariant vector and a contravariant vector. The authors of the book under review make it their purpose to get rid of all extraneous features in the theory of curved spaces and this purpose is very commendable but it must be said at once that in spite of some good ideas (they recognize, for instance, the importance for the theory of what they call homographies, i.e. linear and multilinear vector functions) their attempt results in a *failure*.

The situation may be best characterized by stating that the authors have not succeeded in introducing the most fundamental concept in the theory of curved space—the curvature, or the Riemann, tensor—in an *intrinsic* or absolute way, i.e. without the use of extraneous or arbitrary things. In their attempt to eliminate extraneous things they stopped half way: they got rid of coordinates but instead of studying curved space directly they use a representation of it on a Euclidean space, a representation which, as the authors themselves recognize, involves a certain degree of arbitrariness. But the really strange thing is that because in *their* treatment this tensor is introduced with the aid of notions which have no intrinsic significance the authors conclude that the tensor itself is of no or little importance. On this point (which is the central point in their criticism of the application of geometry of curved space to physics) Burali-Forti and Boggio are behind those geometers who while using coordinates succeed in discriminating as to which expressions have a meaning independent of them. And it must be remarked that, of course, it *is* possible to introduce the Riemann tensor intrinsically and that, in fact, the authors themselves were not so very far from it when they introduced the Riemann curvature. It would be sufficient to notice that the Riemann curvatures at a given point (which they introduce intrinsically) determine completely the Riemann tensor at that point.

Outside of this main line of attack on the relativity theory the authors bring forth against this theory all possible arguments without finding anything to say in its favor. Most of these arguments cannot be taken seriously; to discuss them here would mean to enter into polemics which is not the purpose of this review. It must be said, however, that against some points in the theory of relativity the authors raise objections which are not entirely without justification as, for example, in the case of the postulate according to which all true laws of nature ought to be expressible as covariant relations, especially in the application of this postulate to the energy tensor.

The book cannot be recommended as interesting or instructive reading; in addition to the drawbacks already mentioned it is not self-sufficient as it presupposes the knowledge of the contents of a textbook on vector analysis by Burali-Forti and Marcolongo. Those who specialize in differential geometry might find some worthwhile ideas; besides, the effort of recognizing familiar things under a new form might prove useful as a sort of mental gymnastic. But those who will use it must be warned that it is dangerous to take anything at its face value. To give but one example: the authors state the theorem, not in general true, that if two spaces (of dimension higher than two) have the same metrics one can be obtained from the other by a translation or a symmetry in the containing space — and "prove" it (p. 168).

On the whole, it is to be regretted that such a book has been published. Not so much because it is apt to increase the existing confusion as to the so-called relativity theory (I do not think that it will appreciably increase this confusion because plenty of other books and pamphlets display much the same type of arguments against the relativity theory without displaying so many formulas and are therefore more accessible to the public). It is more to be regretted that some sound ideas and a little sound criticism have been buried among things of doubtful value. But most of all it is to be regretted that it actually may hamper the spreading of the method of direct geometrical analysis in general, and vector analysis in particular.

G. Y. Rainich.

The Geometry of René Descartes. Translated from the French and Latin by David Eugene Smith and Marcia L. Latham. With a facsimile of the first edition, 1637. Chicago, The Open Court Publishing Co., 1925. xiii+246 pages. Price $4.00.

American mathematicians are fortunate in having within their reach through this publication a facsimile of Descartes' original *La Géométrie*, accompanied by a very good translation into English. It is with regret that the announcement was received recently of the death of one of the translators, Marcia L. Latham of Hunter College. Descartes' *Geometry* came to be highly esteemed soon after its appearance in 1637, and to our day it has maintained its place as a classic. It enjoys the distinction of being the book from which Isaac Newton acquired his first knowledge of geometry. And yet one experiences some difficulty in stating, in a few words, exactly wherein the great novelty of the work lies. One cannot say truthfully that Descartes was "the" inventor of analytical geometry, for Fermat about the same time accomplished substantially the same; to be sure, Fermat's treatise did not appear in print until 42 years after that of Descartes. One cannot say that Descartes first employed coordinates, because their use goes back to Greek time. It is not

true that Descartes displayed great inventive power in introducing into algebra our exponential notation for positive integral powers; before him a term like $5a^4$ was written 5*a*4 by Hérigone and $5a^{IV}$ by Hume. It is rather a happy combination of various innovations in one book of compact form, exhibiting the novel geometric use of an equation with more than one unknown, in the solution of difficult applied problems, which gives it celebrity.

FLORIAN CAJORI.

Principii di Filosofia Naturale, Teoria della Gravitazione. By SIR ISAAC NEWTON; translated and edited by FEDERIGO ENRIQUES AND U. FORTI. Rome, A. Stock, 1925. 218 pages. Price, 16 lire.

This is the third volume of the series "Per la Storia e la Filosofia della Matematiche" edited by Professor Enriques. It consists of a translation of Newton's *Philosophiae naturalis principia mathematica*, which appeared in 1687, together with a brief biography of Newton and an appendix of about fifty pages of notes. In this appendix there is given a historical survey of the following topics: (1) The concept of mass or quantity of matter; (2) Force and the laws of motion; (3) Motion, space, and time, absolute and relative; (4) Newton's mechanics as a cosmic science. In the third of these "notes" the historical development of the Einstein theory is sketched.

DAVID EUGENE SMITH.

Il "Metodo" di Archimede e le origine dell' Analisi Infinitesimale nell' Antichità. By ENRICO RUFINI. Rome, A. Stock, 1926. viii+293 pages. Price, 22.50 lire.

This is the fourth volume of the series "Per la Storia e la Filosofia delle Matematiche" edited by Professor Federigo Enriques. It constitutes a monument unconsciously erected to his own memory by the author, who died on November 3, 1924, at the early age of thirty-four, just after completing his manuscript,—a man of whom Professor Enriques speaks very feelingly with respect to his versatility of genius and the breadth of his scholarly interests: "Le Matematiche, le Lettere e la Filosofia lo interessavano quasi in egual grado, e questa varia cultura doveve poi avviarlo assai naturalmente alla storia della scienza." A list of his works was published in December, 1924, in the *Archivio della Storia della Scienza.*

This volume makes a departure from the line followed in the first three numbers of the series, all of which were translations of certain classics, with comments by Professor Enriques or the immediate editor of the volume in question. Dr. Rufini has not been content with translating the Archimedean "Method"; he has introduced it by a "Parte prima" of eighty-eight pages on the origin and development of the infinitesimal analysis in the period closing

with Archimedes. Furthermore, in Part III the author discusses the integrations of Archimedes, devoting nearly a hundred pages to the subject and referring briefly to the influence of the ancients upon the early writers on the calculus in the sixteenth and seventeenth centuries.

The test of the "Method" is based chiefly upon the Kliem translation of Heath's *Archimedes*. There is a brief bibliography of the history of Greek mathematics and a list of the translations of the "Method."

The first part treats of the following topics: (1) The Pythagorean geometry; (2) The critical attitude of Parmenides of Elea; (3) The polemics of Zeno of Elea; (4) The infinitesimal as suggested by Democritus of Abdera; (5) The quadrature of the circle of Antiphon and Bryson (with recognition of but not complete acceptance of the opinion that undue credit has been given to the latter); (6) Eudoxus of Cnidus and the critical (by a curious blunder in the contents, *mitica* (mythical) is used for *critica*) systematizing of the study of the infinitesimal; (7) Eudoxus and the proof by exhaustion; (8) Observations on the method of exhaustion; (9) The infinite and the infinitesimal according to Aristotle; (10) Book XII of Euclid's *Elements*; (11) The work of Archimedes; (12) The mechanical method of Archimedes.

The second part contains the translation of the work in question, and the third relates, as already stated, to Archimedes's integrations, all of which are familiar to non-Italian readers in the translations of Heath, Kliem, and Ver Eecke.

DAVID EUGENE SMITH.

ARTICLES IN CURRENT PERIODICALS

American Journal of Mathematics, volume 48, no. 2, April, 1926: "The theory of the binary octavic" by A. M. Whelan, 73–100; "The Borel summability of Fourier series" by M. H. Stone, 101–112; "Self dual space curves" by T. R. Hollcroft, 113–124; "Expansions in terms of certain polynomials connected with the gamma-function" by B. P. Hoover, 125–138, "On the value of the Naperian base" by D. H. Lehmer, 139–143; "On differential inversive geometry" by F. Morley, 144–146. No. 3, July, 1926: "On the structure of a continuum, limited and irreducible between two points" by A. Wilson, 147–168; "The expansion problem for a certain system of ordinary linear second order differential equations" by M. G. Carman, 169–182; "The distribution of lift over thin wing sections" by C. A. Shook, 183–203; "The correspondence between the tangent plane of a surface and its point of contact" by E. P. Lane, 204–214; "On multiple iterated integrals" by H. J. Ettlinger, 215-222; "Cubic involutions and a C_4" by H. S. White, 223–224.

Annals of Mathematics, second series, volume 27, no. 3, March 1926: "Transformation of the Kummer criteria in connection with Fermat's last theorem" by H. S. Vandiver, 171–176; "Algebraic potential curves" by J. L. Coolidge, 177–186; "A method of deriving the infinite double products in the theory of elliptic functions from the multiplication theorems" by G. Mittag-Leffler, 187–194; "Miscellaneous questions in the theory of differential equations. 1. On the method of Frobenius" by E. Hille, 195–198; "Non-measurable functions connected with certain functional equations" by H. Blumberg, 199–208; "Entire functions defined by certain power series" by J. T. Colpitts, 209–223; "Approximation of curves and surfaces by algebraic curves and surfaces" by P. A. Smith, 224–244; "Note on matrices in a given field" by J. H. M. Wedderburn, 245–248; "On the arithmetical applications of the

power series for theta quotients" by E. T. Bell, 249–257; "Conditions for self dual curves" by T. R. Hollcroft, 258–270; "Semi-parallel maps of surfaces" by W. C. Graustein, 271–278; "Projective invariants of affine geometry of paths" by O. Veblen and J. M. Thomas, 279–296.

Bulletin of the American Mathematical Society, volume 32, no. 2, March–April 1926: "Properties of unrestricted real functions" by H. Blumberg, 132–148; "On the constructions of factor stencils" by D. N. Lehmer, 149–150; "Note on the second law of the mean for integrals" by J. Tamarkin and C. E. Wilder, 151–152; "On a certain functional condition" by J. P. Ballantine, 153–155; "On the correspondence between space sextic curves and plane quartics in four-space" by B. C. Wong, 156–158; "On irredundant sets of postulates" by H. M. Gehman, 159–161; "On the solution of higher degree algebraic equations" by G. James, 162–164; "The conditions for a fixed point in projective differential geometry" by A. L. Nelson, 165–168. No. 3, May–June, 1926: "The Gibbs lecture for 1925: Some modern views of space" by J. Pierpont, 225–258; "Note on a problem in approximation with auxiliary conditions" by D. Jackson, 259–262; "An approximation to the least root of a cubic equation with application to the determination of units in pure cubic fields" by T. A. Pierce, 263–268; "Notes on the rational plane cubic curve" by T. W. Moore, 269–274; "Covering theorems" by R. L. Moore, 275–281; "The convergence of a general mean of measurements to the true value" by E. L. Dodd, 282; "Consecutive quadratic residues" by A. A. Bennett, 283–284; "Note on a pair of properties which characterize continuous functions" by C. H. Rowe, 285–287. No. 4, July–August, 1926: "A boundary value problem in the calculus of variations" by G. A. Bliss, 317–330; "A connected and regular point set which contains no arc" by R. L. Moore, 331–332; "The alternation of nodes of linearly independent solutions of second order difference equations" by O. Dunkel, 333–334; "A notion of uniform integrability" by R. E. Langer and J. D. Tamarkin, 335–337; "A theorem on connected point sets which are connected im kleinen" by R. L. Wilder, 338–340; "A ray of numerical functions of r arguments" by E. T. Bell, 341–345; "Approximate solutions of a system of differential equations of mathematical physics by least squares", by N. Kryloff, 346–350; "Note on a theorem of Kempner concerning transcendental numbers" by H. Blumberg, 351–356; "An invariant of a general transformation of surfaces" by W. C. Graustein, 357–364; "Wilczynski's and Fubini's canonical systems of differential equations" by E. P. Lane, 365–373; "Representation of integers by certain ternary cubic forms" by F. S. Nowlan, 374–380.

Journal of Mathematics and Physics, M. I. T., volume 5, no. 4, June 1926: "Grassmannian geometry in Riemannian space" by C. L. E. Moore, 191–200; "The fundamental theorem of almost periodic functions of two variables" by P. Franklin, 201–235; "A correction" by P. Franklin, 236–237; "Compounds of Scott product determinants" by L. H. Rice, 238–250; "Introduction to the relativistic interpretation of the quantum theory" by T. De Donder, 251–261.

Proceedings of the National Academy of Sciences, volume 12, no. 6, June 1926: "Conformal invariants" by J. M. Thomas, 389–392; "Generalization of Kronecker's relation among the minors of a symmetric determinant" by H. Barton, 393–396; "On the classical Dirichlet problem for general domains" by O. D. Kellogg, 397–405. No. 7, July 1926: "Form of the number of the prime power subgroups of an abelian group" by G. A. Miller, 470–472.

Tohoku Mathematical Journal, volume 26, nos. 3, 4, April 1926: "Historical note on the solution of an algebraic equation" by G. A. Miller, 384–390; "B, E polynomials and their related integrals" by E. T. Bell, 391–405.

Transactions of the American Mathematical Society, volume 28, no. 2, April 1926: "New division algebras" by L. E. Dickson, 207–234; "The first and second variations of a double integral for the case of variable limits" by H. A. Simmons, 235–251; "On extending a continuous (1–1) correspondence of two plane continuous curves to a correspondence of their planes" by H. M. Gehman, 252–265; "Systems of equations in an infinity of unknowns, whose solution involves an arbitrary parameter" by I. M. Sheffer, 266–286; "Solution of certain functional equations relative to a general linear set" by M. H. Ingraham, 287–300; "Combinatorial analysis situs" by J. W. Alexander, 301–329; "Geometries of paths for which the equations of the paths admit $n(n+1)/2$ independent linear first integrals" by L. P. Eisenhart, 330–338; "Multiply transitive substitution groups" by G. A. Miller, 339–345; "An extension of Lagrange's expansion" by H. Bateman, 346–356.

UNDERGRADUATE MATHEMATICS CLUBS

EDITED BY H. J. ETTLINGER, 3110 Harris Park Ave., Austin, Texas

CLUB ACTIVITIES

GAMMA ETA MU, University of Redlands, Redlands, California.

At the suggestion of advanced students, an organization known as the Gamma Eta Mu Club was formed at the University of Redlands on November 13, 1924, for the purpose of promoting interest in mathematical subjects. Membership was at first limited to those majoring or minoring in mathematics, but on March 1, 1926 was extended to include those majoring in physics or engineering. The following programs have been given:

December 8, 1924. "Pre-Greek history of mathematics" by Kate Smallin; "Inscribed and circumscribed circles in a parabolic segment" by Jack Boren; "Construction of regular decagon" by Lewis Hammen.

January 19, 1925. Picnic at Camp Kill Kare.

February 16. "Greek history of mathematics" by Sarah Russell; "Ladder and post problem" by Roy Slocum.

March 16. "Hindu mathematics" by Alexis Maredudin; "Four common tangents to two circles" by Julius Williams.

April 20. "Twelve eminent mathematicians" by William Garner and Nelson Painter; "Graduate work in mathematics" by Leslie Hosegood; "Magic squares" by Doris Smith.

May 18. "Life of Newton" by Walter Burger; "Nine point circle" by Marion Scott; "Fourth dimension" by Mrs. Ethyl White.

May 25. Picnic at Plunge Creek.

October 14. Picnic in Mill Creek Canyon.

October 26. "History of π" by Helen Irwin; "Evolute of $x=(t^2+1)/4$, $y=t^3/6$" by James Wright.

November 16. "Life of Euclid" by Fred Riedman; "Maximum chord of cardioid" by Joseph Hartshorn.

December 14. "Babylonian mathematics" by Alexis Maradudin; "Locus of center of circle cutting three given circles at a constant angle" by Mrs. Ethyl White.

January 18, 1926. "Fermat's last theorem" by George Shinn; "A particle falling where resistance is proportional to the square of velocity" by Jack Boren.

February 15. "Curves satisfying condition $s^2=4ky$, where s is the length of arc" by William Richardson.

March 16. Lecture and demonstration of Willys Knight motor by the company's representative.

April 19. "San Bernardino's water system" by Leslie Hosegood; "Construction of circle tangent to three other circles" by Clara Seaton.

May 19. Picnic at Plunge Creek.

The officers were: President, William Richardson; vice-president, Clara Seaton; Secretary-treasurer, Mrs. Ethyl White; faculty advisers, Professor O. W. Albert, mathematics, and Professor H. E. Marsh, physics.

(Report by Professor Albert)

CENTENARY COLLEGE MATHEMATICS CLUB, Shreveport, La.

In the early part of October, 1925, those students of Centenary College who had a special interest in mathematics were called together and a Mathematics Club was organized. Those instrumental in organizing the club were Miss Velinsky and Dean Hardin of the mathematics department and Dr. I. Maizlish, head of the physics department. The officers elected are:

Emmet Meadows, President; Miss Emily Dean Odom, Secretary-treasurer.

The Club meets monthly. Some of the topics thus far discussed are:

"The story of Pi", Emmet Meadows;

"Discovery of Neptune", Leon Scales;

"Life and works of Willard Gibbs", W. G. Banks;

"Practical mathematics", Mr. Anderson;

"History of arithmetic", Clingman Munday;
"Value of mathematics", Dean John A. Hardin.

In his closing address Dean Hardin expressed his pleasure at the rapid progress of the Club, which had an average attendance of fifteen for the year. He stated that the prospect was very bright for an even more successful organization next year.

(Report by Emily Dean Odom, secretary-treasurer)

UNDERGRADUATE MATHEMATICS CLUB, University of Iowa, Iowa City, Iowa.

[*1925*, 432]

Altogether thirteen meetings were held during the past year. A consistent effort was made to provide interesting programs and the result was an average attendance of fifty. A new feature was a short social time at the beginning of each meeting during which tea and cakes were served. In the latter half of the year we tried particularly to interest Freshmen and the attendance at one special meeting was ninety-nine. The subjects discussed and the speakers follow: The measurement of angles, Dr. Wilson; Complex numbers, Miss Reger and Miss Peters; Computation of Pi, Mr. Stehn; Partial fractions, Mr. Wilson and Mr. Long; The why of the number e, Mr. Briggs; The circles of Appolonius, Mrs. Wilson; Twenty-five years in mathematics, Prof. Rietz; Trisection of angles, Miss Reger; Russian peasant method of calculation, Mr. Allen; Number systems of the ancients, Mr. Messick; Factorials and binomial coefficients, Dr. Woods. An additional feature was the presentation at each meeting of a problem for solution at the following meeting.

(Report by Miss Leona Schuster, secretary)

JOHNS HOPKINS MATHEMATICAL CLUB, Baltimore, Md.

[*1921*, 181]

The Undergraduate Mathematics Club at the Johns Hopkins University met bi-weekly during 1925–6.

The following subjects were presented:

1. "The game 'Nim'," Louis M. Volansky.
2. "Possible and impossible 'Euclidean' constructions," Dr. A. Cohen.
3. "Problems in probability," Walter S. Dawkins.
4. "History of π," Brainerd D. Wilson.
5. "Magic squares," David M. Ashkenaz.
6. "The nine-point circle," Louis M. Volansky.
7. "Non-Euclidean geometry," W. S. Dawkins.
8. "Geometrical paradoxes," Victor R. Deitz.
9. "Geometrical representation of complex numbers," Dr. A. Cohen.

(Report by Professor A. Cohen)

WHITE MATHEMATICS CLUB, University of Kentucky, Lexington, Ky.

[*1925*, 430]

Sept. 23, 1925. Election of the following officers:
President, D. E. South; Secretary, M. C. Brown. *Essentials of Mental Measurements* by Brown and Thompson was adopted as a basis for the year's club work.
Oct. 15. "Curiosities" by Professor H. H. Downing.
Oct. 27. "Suggestions for programs" by Professor J. Morton Davis.
Nov. 19. "Elementary theory of probability" by D. E. South, instr.
Dec. 3. "Psycophysical Methods" by Professor E. L. Rees.
Dec. 17. "Difference thresholds" by Dr. F. E. LeStourgeon.
Jan. 14, 1926. "Introduction to correlation" by Prof. J. Morton Davis.
Feb. 4. "Coefficient of correlation" by Mr. M. C. Brown, instructor.
Feb. 18. "Correlation ratio" by Mr. D. O. Streyffeler, instructor.

Mar. 4. "Study of grades made by freshmen in mathematics and in mental tests at U. of K. in 1925" by Mr. E. J. Canaday, instructor.
Mar. 18. "Methods of computing correlation" by Professor P. L. Boynton of the Psychology Department.
April 18. "Types of collineations in the ternary field" by Miss Vada Lee Nelson, graduate student.
April 23. "Circular inversions" by Mr. H. W. Mobley, grad. asst.
May 13. "Irrational numbers" by Miss Mary Hester Cooper, grad. asst.

(Report by M. C. Brown, secretary)

The Mathematics Club of the University of Kansas, Lawrence, Kansas

The officers of the Mathematics Club of the University of Kansas for the year 1925-26 were: President, Elizabeth Bolinger '26; Vice President, Lloyd Young '26; Secretary-Treasurer, Vera Bolton '26.

The following topics were presented at the meetings:

October 5, 1925: "Some new properties of determinants" by Professor E. B. Stouffer

October 19: "Systems of conics through four fixed points" by H. K. Hughes, Gr.

November 2: "The slide rule for complex numbers" by Professor M. E. Rice.

November 16: "Reminiscences of Benjamin Peirce" by Marjory Council '26.

December 7: Play "Flatlanders" by members of the club. "The fourth dimension" by Professor R. H. Wheeler.

January 11, 1926: "Geometrical representations of indeterminate forms" by Grace Poe '26.

February 1: "The nine point circle" by C. A. Reagan, Gr.

February 15: "1925 as a centennial year in mathematics" by Helen McFerren '26.

March 1: "Mathematical poems" by Helen Mark '25. "Perpetual calendars" by Rose Middlekauff '26.

March 15: "Some theorems of geometrical optics" by Lloyd Young '26.

April 19: "Some magic in mathematics" by Zella Colvin, Gr.

May 2: "Algebraic curves related to conics" by P. F. Wall, Gr.

May 17: Annual picnic.

(Report by Vera Bolton)

The Napierian Club of De Pauw University, Greencastle, Indiana

The officers for the year 1925-1926 were: President, James V. Brown, '26; Vice-President, Ruth Bickel, '26; Secretary, Gertrude Hendrix, '26; Treasurer, Orin Sykes, '26. Monthly meetings were held throughout the year, and the programs were as follows:

October 14, 1925: Selection of new members.

November 11, 1925: "The Life and Work of John Napier" Elizabeth Erwin, '26.

December 9, 1925: "Einstein's Theory of the Fourth Dimension" by Harold Weeks, '26.

January 13, 1926: "Mathematics and Mathematicians in Music" by Professor H. E. H. Greenleaf.

February 24, 1926: "The Slide Rule" by Professor Clark Arnold.

March 17, 1926: A debate on the question, "Resolved that ten hours of mathematics should be required for graduation at De Pauw University." Decision negative.

April 15, 1926: "Chinese Mathematics" by Horace Yu, '26.

May 27, 1926: Alumni letters. Election of officers.

(Report by Gertrude Hendrix, secretary)

The Agnesi Club of Agnes Scott College, Decatur, Ga.

The program for the year 1925-1926 was the following:

October 6, 1925. Introductory talk by the president giving organization, purpose, and plans of the Agnesi Club. The organization took place in 1921 and the purpose was to stimulate the interest in physics, mathematics and astronomy.

November 3. "Mechanisms, planimeters, and integraphs" by Professor W. W. Rankin.

December 1. Astronomical program. "Nebulae and comets" by Miss F. Swann, '26 and Miss E. Kennedy, 26, with slides by Miss Emily E. Howson, professor of physics.

February 4, 1926. Catch problems in mathematics by Miss L. S. Wallace, '26. "History of the calculus" by Miss M. E. Hammond, '26.

March 19. "Theory of equations" by Professor L. E. Dickson, University of Chicago. Social hour.

April 6. Physics program on transverse waves. "Gamma rays" by Miss K. Pitman, '26. "X-rays" by Miss E. Lynn, '27. "Ultra violet and visible waves" by Miss H. Huff, '26. "Infra-red waves" by Miss V. B. Grant, '27. These talks were accompanied by slides and experiments. Radio program.

May 4. Mathematics play "The Eternal Triangle" written by Miss S. Slaughter, '26, and Miss F. Swann, '26, read by Miss Slaughter.

(Report by Miss Lynn, Secretary)

THE MATHEMATICS CLUB of the College of the City of Detroit, Detroit, Mich.

The Mathematical Club of the College of the City of Detroit was organized January 18, 1926 with fifteen charter members. The following officers were elected:

President, Max Coral, '27; secretary, Frances Garvey, '27; program committee, Philip Gentile, '26 and Kenneth Salisbury, '28.

The papers presented during the first semester of the club's existence were:

January 18, 1926. "Non-euclidean geometry" by Professor J. W. Baldwin.

March 24. "Mathematical recreations" by David Koretz, '26.

April 14. "Graphical solution of equations" by Professor T. R. Running, University of Michigan.

May 18. "Three famous problems" by Rose Chesluk, '27.

(Report by Frances Garvey, Secretary)

PROBLEMS AND SOLUTIONS

EDITED BY B. F. FINKEL, OTTO DUNKEL, AND H. L. OLSON

Send all communications about Problems and Solutions to B. F. Finkel, Springfield, Mo. All mauscripts should be typewritten, with double spacing and with a margin at least one inch wide on the left.

PROBLEMS FOR SOLUTION

[N. B. Problems containing results believed to be new, or extensions of old results are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known textbooks or results found in readily accessible sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist members of the Association with their difficulties in the solution of such problems.]

3226. Proposed by H. E. Trefethen, Colby College.

The cross section of a circular ring is crescent shaped. The outer arc is a semicircle. The inner arc has a radius of one inch and its center is at the center of the ring. Find the greatest possible volume for the ring.

3227. Proposed by H. W. Reddick, Cooper Union Institute of Technology.

Solve the differential equation

$$\frac{d^5y}{dx^5} - \frac{dy}{dx} - \frac{4y}{x} = 0.$$

3228. Proposed by Nathan Altshiller-Court, University of Oklahoma.

The perpendiculars dropped from the vertices of a triangle upon the lines joining the mid-points of the opposite sides to the orthocenter of the triangle meet the respective sides of the triangle in three collinear points.

The line joining these three points is perpendicular to the Euler line of the triangle.

3229. Proposed by John Biggerstaff, Seattle, Washington.

Prove that the function $\{p(u)\ p'(u)+p^2(u)-1\}$ has five zeros u_1, u_2, u_3, u_4, u_5, in a period parallelogram such that

$$\sum_{n=1}^{n=5} u_n = 2\lambda\omega_1 + 2\mu\omega_2,$$

where λ and μ are integers and $p(u)$, Weierstrass's elliptic function.

Verify that if $z=p(u)$, these values of u give the five roots of the equation $4z^5-z^4-g_2z^3+(2-g_3)\ z^2-1=0$, g_2 and g_3 being constants and solve the general quintic by this method.

3230. Proposed by C. N. Schmall, New York City.

Determine the point in an ellipse at which the two focal distances include the greatest angle.

3231. Proposed by R. B. Stone, Purdue University.

The root-mean-square (R. M. S.) of n numbers $x_1, x_2, \cdots, x_n$ is defined by the formula

$$R.M.S. = \left(\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n}\right)^{1/2}.$$

(*a*) For what values of n is the R. M. S. of the first n integers also an integer?

(*b*) For what values of a and n is the R. M. S. of n successive integers

$$a,\quad a+1, \cdots\ a+n-1$$

also an integer?

(*c*) Under what conditions is the R. M. S. of n integers also an integer?

SOLUTIONS

2672 [1918, 74; 1925, 385]. Proposed by E. T. Bell, Seattle, Washington.

There is an identity in z, (1) $A(z) \equiv B(z)\ C(z)$; e.g., $A(z) = 1/(1-k^2z^2)$; $B(z) = 1/(1-kz)$; $C(z) = 1/(1+kz)$; and the formal expansions $A(z) = \Sigma a(n)z^n$, $B(z) = \Sigma b(n)z^n$, $C(z) = \Sigma c(n)z^n$, $(n = 0, 1, \cdots, \infty)$, when substituted in (1), give, on equating coefficients (2):

$$a(n) = b(n)c(0) + b(n-1)c(1) + \cdots + b(0)c(n).$$

If (2) is an identity in n, justify such a use of non-convergent series to obtain it (*e.g.*, for $|k| \geqq 1$ in the above). This method of finding important identities (2) has been freely used by Hermite and many others without question of its validity, and without offering independent proofs of (2).

Solution by Otto Dunkel, Washington University, St. Louis, Mo.

By the formal expansion of $f(z)$ in a power series in z we mean ordinarily the rule given by the Maclaurin series. Hence when we say that each of the three functions $A(z), B(z), C(z)$ may be expanded as in the problem, it is implied that all of their derivatives exist in the neighborhood of $z=0$. Consequently, since $A(z) \equiv B(z)\ C(z)$, we must have $A^{(n)}(0) = [B(0)C(0)]^{(n)}$, where the index n indicates the nth derivative. The process for the development of the right side in terms of the derivatives of B and C is purely formal and it is unique. To obtain this formal development, we may at first suppose that each of the three functions admit formal developments which are convergent and which represent them in value in the neighborhood of $z = 0$. Then since

$$\sum \frac{A^{(i)}(0)}{i!} z^i = \sum \frac{B^{(i)}(0)}{i!} z^i \sum \frac{C^{(i)}(0)}{i!} z^i,$$

we must have

$$\frac{A^{(n)}(0)}{n!} = \sum_{i=0}^{n} \frac{B^{(i)}(0)}{i!} \frac{C^{(n-i)}(0)}{(n-i)!}.$$

This gives the form of the development desired and it must be true whether the series converge or not. This proves the theorem. The result is Leibniz' theorem, which may be established directly by other known methods, and then this theorem may be used to establish the desired result without the use of the series.

Also solved by HARRY LANGMAN.

3151 [3147; 1925, 434]. Proposed by J. A. Bullard, U. S. Naval Academy.

The cylinder $(x/a)^2 + (y/b)^2 = 1$, passes through the sphere, $x^2 + y^2 + z^2 = a^2$. If e is the eccentricity of the ellipse $(x/a)^2 + (y/b)^2 = 1$, show that

(*a*) the surface cut from the sphere $= 8a^2 \arccos e$,

(*b*) the volume cut from the sphere $= 8a^3[e(1 - e^2)^{1/2} + \arccos e]/3$.

SOLUTION BY J. M. EARL, University of Minnesota.

(*a*) The width of an elementary zone cut out by the planes $x = x$ and $x = x + dx$ is

$$ds = \sqrt{1 + \left(\frac{dz}{dx}\right)^2}\,dx \text{ where } x^2 + z^2 = a^2. \text{ Hence } ds = \frac{a\,dx}{(a^2 - x^2)^{1/2}}.$$

The element of arc along this zone is

$$ds' = \sqrt{1 + \left(\frac{dz}{dy}\right)^2}\,dy \text{ where } y^2 + z^2 = (a^2 - x^2). \text{ Hence } ds' = \frac{(a^2-x^2)^{1/2}dy}{[(a^2 - x^2) - y^2]^{1/2}}.$$

The required area, which is 8 times the area cut off in the first octant, is

$$A = 8\iint ds\,ds' = 8a\int_{x=0}^{x=a} dx \int_{y=0}^{y=(b/a)(a^2-x^2)^{1/2}} \frac{dy}{[(a^2-x^2)-y^2]^{1/2}} = 8a^2 \arccos e.$$

(*b*) In a similar manner, the required volume is

$$\begin{aligned} V &= 8\int_{x=0}^{x=a} \int_{y=0}^{y=(b/a)(a^2-x^2)^{1/2}} z\,dx\,dy = 8\int_{x=0}^{x=a} dx \int_{y=0}^{y=(b/a)(a^2-x^2)^{1/2}} [(a^2 - x^2) - y^2]^{1/2}\,dy \\ &= (8a^3/3)[(b/a^2)(a^2 - b^2)^{1/2} + \arcsin(b/a)] \\ &= (8a^3/3)[e(1 - e^2)^{1/2} + \arccos e]. \end{aligned}$$

Also solved by E. T. ALLEN, MICHAEL GOLDBERG, E. C. KIEFER, J. B. REYNOLDS, H. A. SIMMONS, and R. H. SCIOBERETI.

3152 [3148; 1925, 480]. Proposed by E. M. Berry, Purdue University.

Let Δ be the determinant $|x_1y_2z_3| \neq 0$, and let X_i be the cofactor of x_i in Δ and Y_i the cofactor of y_i, etc. Prove that if

$$\begin{vmatrix} 1/x_1 & 1/x_2 & 1/x_3 \\ 1/y_1 & 1/y_2 & 1/y_3 \\ 1/z_1 & 1/z_2 & 1/z_3 \end{vmatrix} = 0, \text{ then } \begin{vmatrix} 1/X_1 & 1/X_2 & 1/X_3 \\ 1/Y_1 & 1/Y_2 & 1/Y_3 \\ 1/Z_1 & 1/Z_2 & 1/Z_3 \end{vmatrix} = 0$$

and conversely.

SOLUTION BY H. A. SIMMONS, Northwestern University.

Denote the second determinant by Δ' and the third by D'. Then on expanding these determinants we find

$$\begin{aligned} \Delta' &= \lambda_1[x_2x_3y_1z_1X_1 + x_3x_1y_2z_2X_2 + x_1x_2y_3z_3X_3], \\ D' &= \Delta\lambda_3[X_2X_3Y_1Z_1x_1 + X_3X_1Y_2Z_2x_2 + X_1X_2Y_3Z_3x_3], \end{aligned} \tag{1}$$

where λ_1 and λ_2 are two finite factors, each different from zero—these facts being implied by the forms in which the determinants Δ' and D' are given. Now since

$$\sum_{i=1}^{3} z_i X_i = 0 \text{ and } \sum_{i=1}^{3} Z_i x_i = 0,$$

by a well known, elementary property of determinants, we can write (1) in the form

$$\Delta' = -\lambda_1(x_3z_2X_2Z_3 - x_2z_3X_3Z_2), \qquad D' = -\Delta^2\lambda_2(x_3z_2X_2Z_3 - x_2z_3X_3Z_2). \tag{2}$$

Since $\Delta \neq 0$ by hypothesis and λ_1, λ_2 do not vanish in this problem, we see that $\Delta' = 0$ if, and only if, $D' = 0$, which proves the theorem and its converse.

Also solved by HARRY LANGMAN, and J. J. NASSAU.

3153 [3149; 1925, 481]. Proposed by A. A. Bennett, Lehigh University.

Given a circle, C. Inscribe in C a simple quadrilateral whose pairs of non-adjacent sides produced meet in points P and Q respectively. Show that the circles with P and Q as centers and orthogonal to C meet in a point R, where PRQ is a right angle.

SOLUTION BY NATHAN ALTSHILLER-COURT, University of Oklahoma.

The points P, Q are two diagonal points of the complete quadrangle having the same vertices as the simple quadrilateral inscribed in the given circle C, therefore the points P, Q are conjugate with respect to C, and the polar of P with respect to C is the perpendicular from Q to the line PO joining P to the center O of C, the foot S of this perpendicular being the inverse of P with respect to C. The circle (L) having PQ for diameter will pass through S, and therefore be orthogonal[1] to C.

We have thus three circles orthogonal to C: the circle (L) and the two circles (P), (Q), having for their centers the points P, Q. Since the centers of these three circles are collinear, the three circles are coaxal[2], i.e., the common points R, R' of the circles (P), (Q) lie on the circle (L), hence PRQ is a right angle.

REMARK. We have thus incidentally proved the proposition:

If two circles are orthogonal to a third circle and their centers are conjugate with respect to that circle, the two circles are orthogonal to each other.

Also solved by J. W. CLAWSON, RUFUS CRANE, W. J. PATTERSON, J. ROSENBAUM and MABEL YOUNG.

3154 [3150; 1925, 481]. Proposed by A. S. Wiener, Cornell University.

Prove

$$\begin{vmatrix} b^2+c^2 & 0 & -cd & be & 0 \\ 0 & a^2+d^2 & 0 & ab & ad \\ -cd & 0 & d^2+e^2 & 0 & ce \\ be & ab & 0 & b^2+e^2 & 0 \\ 0 & ad & ce & 0 & a^2+c^2 \end{vmatrix} \times \begin{vmatrix} b^2+c^2 & 0 & cd & be & 0 \\ 0 & a^2+d^2 & 0 & ab & ad \\ cd & 0 & d^2+e^2 & 0 & ce \\ be & ab & 0 & b^2+e^2 & 0 \\ 0 & ad & ce & 0 & a^2+c^2 \end{vmatrix}$$

$$= \begin{vmatrix} b^4+c^4 & 0 & -c^2d^2 & b^2e^2 & 0 \\ 0 & a^4+d^4 & 0 & a^2b^2 & a^2d^2 \\ -c^2d^2 & 0 & d^4+e^4 & 0 & c^2e^2 \\ b^2e^2 & a^2b^2 & 0 & b^4+e^4 & 0 \\ 0 & a^2d^2 & c^2e^2 & 0 & a^4+c^4 \end{vmatrix}.$$

[1] Nathan Altshiller-Court, *College Geometry*, p. 147, Johnson Publishing Co., Richmond, Va., 1925.
[2] *Ibid*, p. 188.

SOLUTION BY HARRY LANGMAN, Brooklyn, N. Y.

If we square the determinant

$$\begin{vmatrix} 0 & b & \mp c & 0 & 0 \\ a & 0 & 0 & d & 0 \\ 0 & 0 & d & 0 & e \\ b & e & 0 & 0 & 0 \\ 0 & 0 & 0 & a & c \end{vmatrix}$$

we obtain the first or second determinant of the problem, according as the upper or lower sign before c is taken. Expanding this according to the minors of the first two columns, we obtain at once

$$c(b^2d^2 \mp a^2e^2).$$

Replacing each letter in this equality by its square (choosing the upper sign) and squaring, we obtain the value of the third determinant. The identity is then evident.

3155 [3151; 1925, 481]. Proposed by C. C. Wylie, University of Iowa.

In a concave-convex lens the radii of curvature of the convex and concave surfaces are r_1 and $r_2 > r_1$. Determine the thickness and diameter of the lens so that the centre of mass shall be in the concave surface.

(Moulton's *Celestial Mechanics*, new edition, p. 29)

SOLUTION BY J. L. RILEY, Ouachita College, Arkadelphia, Ark.

The equations of the circles may be expressed in the forms:

$$y^2 = -x^2 + 2r_1x \quad \text{and} \quad y^2 = -x^2 - t^2 + 2r_2x + 2tx - 2r_2t$$

where t is the thickness of the lens.

The centroid of the lens is

$$\bar{x} = \frac{\displaystyle\int_0^\alpha (2r_1x^2 - x^3)dx - \int_t^\alpha (2r_2x^2 + 2tx^2 - x^3 - 2r_2tx - t^2x)dx}{\displaystyle\int_0^\alpha (2r_1x - x^2)dx - \int_t^\alpha (2r_2x + 2tx - x^2 - 2r_2t - t^2)dx}$$

where

$$\alpha = \frac{t(t+2r_2)}{2(r_2 - r_1 + t)}.$$

Remembering that $\bar{x}$ must equal t, we get

$$1 = \frac{\dfrac{(t+2r_2)^3}{(r_2 - r_1 + t)^2} - 2t - 8r_2}{\dfrac{6(t+2r_2)^2}{r_2 - r_1 + t} - 8t - 24r_2}.$$

Hence

$$t^3 + (-6r_1 + 4r_2)t^2 + (6r_1^2 - 20r_1r_2 + 2r_2^2)t + 16r_1^2r_2 - 8r_1r_2^2 = 0.$$

Since t is small, we neglect the terms in t^3 and t^2, and find that, approximately,

$$t = \frac{4r_1r_2^2 - 8r_1^2r_2}{3r_1^2 - 10r_1r_2 + r_2^2}.$$

3159 [3155; 1925, 520]. Proposed by R. E. Moritz, University of Washington.

Show that

$$\frac{d^n}{dx^n}\left[x^{m+n}(\log x)^n\right] = n!x^m\left[1 + {}_n^m p_1 \log x + {}_n^m p_2 \frac{(\log x)^2}{2!} + \cdots {}_n^m p_n \frac{(\log x)^n}{n!}\right],$$

where ${}_n^m p_k$ denotes the sum of the products of the numbers $m+1, m+2, \cdots, m+n$, taken k at a time.

I. Solution by M. S. Knebelman, Princeton University.

Let

$$x = e^v \text{ ; then } \frac{d}{dx} = e^{-v}\frac{d}{dy} = e^{-v}D, \text{ where } D, \equiv \frac{d}{dy}.$$

With this substitution, we get

$$\frac{d^n}{dx^n}\left[x^{m+n}(\log x)^n\right] = (e^{-v}D)^n\left[e^{(m+n)v}y^n\right].$$

To evaluate the last expression, we have two well known theorems on the symbolic operator D:

$$(e^{-v}D)^n = e^{-nv}\prod_{\alpha=1}^{n}(D - n + \alpha) \tag{1}$$

$$f(D)\left[e^{av}F(y)\right] = e^{av}f(D + a)\left[F(y)\right]. \tag{2}$$

By (1), our expression becomes

$$e^{-nv}\prod_{\alpha=1}^{n}(D - n + \alpha)\left[e^{(m+n)v}y^n\right]$$

and by (2) it further reduces to

$$e^{mv}\prod_{\alpha=1}^{n}(D + m + \alpha)\left[y^n\right].$$

Now

$$\prod_{\alpha=1}^{n}(D + m + \alpha)$$

is a polynomial in D; it is of the nth degree and its coefficients are of the form ${}_n^m p_k$, $({}_n^m p_0 \equiv 1)$. We may therefore write

$$\prod_{\alpha=1}^{n}(D + m + \alpha) = \sum_{k=0}^{n} {}_n^m p_k D^{n-k}.$$

Again

$$D^{n-k}\left[y^n\right] = n(n-1)(n-2)\cdots(k+1)y^k = \frac{n!}{k!}y^k;$$

therefore,

$$(e^{-v}D)^n\left[e^{(m+n)v}y^n\right] = n!e^{mv}\sum_{k=0}^{n} {}_n^m p_k \frac{y^k}{k!}$$

and since $y = \log x$ we finally have

$$\frac{d^n}{dx^n}\left[x^{m+n}(\log x)^n\right] = n!x^m\sum_{k=0}^{n} {}_n^m p_k \frac{(\log x)^k}{k!}.$$

II. Solution by the Proposer.

By Maclaurin's theorem

$$x^{m+n+k} = x^{m+n}\left[1 + k\log x + \frac{k^2(\log x)^2}{2!} + \cdots + \frac{k^n(\log x)^n}{n!} + \cdots\right] \tag{1}$$

Differentiating with respect to x, we have, the series on the right being uniformly convergent for all values of $x \neq 0$,

$$(m + n + k)x^{m+n+k-1} = (m + n + k)x^{m+n-1}\left[1 + k\log x + \cdots + \frac{k^n(\log x)^n}{n!} + \cdots\right]$$

$$= \frac{d}{dx}(x^{m+n}) + k\frac{d}{dx}(x^{m+n}\log x) + \cdots + \frac{k^n d}{n!dx}\left[x^{m+n}(\log x)^n\right] + \cdots$$

Similarly, on differentiating both sides of [1] n times with respect to x,

$$(m+n+k)(m+n+k-1)\cdots(m+k+1)x^m\left[1+k\log x+\cdots+\frac{k^n(\log x)^n}{n!}+\cdots\right]$$

$$=\frac{d^n}{dx^n}(x^{m+n})+k\frac{d^n}{dx^n}(x^{m+n}\log x)+\cdots+\frac{k^nd^n}{n!dx^n}[x^{m+n}(\log x)^n]+\cdots \qquad (2)$$

Now

$$(m+n+k)(m+n+k-1)\cdots(m+k+1)=k^n+{}_n^mp_1k^{n-1}+{}_n^mp_2k^{n-2}+\cdots+{}_n^mp_{n-1}k+{}_n^mp_n.$$

hence, on comparing coefficients of k^n in [2] and multiplying the result by $n!$ we obtain the desired result.

Also solved by THEODORE BENNETT.

NOTES AND NEWS

Readers are invited to contribute to the general interest of this department by sending items to H. W. Kuhn, Ohio State University, Columbus, Ohio.

The New York Academy of Sciences offers a prize (the A. Cressy Morrison Prize) to be awarded in December, 1926, for an essay on the intra-atomic activity of the sun.

Kenyon College has conferred the honorary degree of doctor of laws on Professor M. T. PUPIN of Columbia University.

Johns Hopkins University celebrated its fiftieth anniversary on October 22–23, 1926. Among the alumni who delivered addresses on this occasion were Professor A. B. COBLE, of the University of Illinois, and L. P. EISENHART, of Princeton University.

The Mathematical Association was represented by Professor W. L. PORTER at the semi-centennial of the founding of the Agricultural and Mechanical College of Texas October 14–16, 1926; by Professor W. L. MILNE at the inauguration of Doctor Arnold Bennett Hall as president and the semi-centennial of the founding of the University of Oregon October 18–23, 1926; and by Professor ANNA H. PALMIÉ at the hundredth anniversary of Western Reserve University November 12–13, 1926.

Professor L. D. AMES, of the Texas Technological College, has been appointed professor of mathematics at the University of Southern California.

Mr. C. F. BARR, of Purdue University, has been appointed assistant professor of mathematics at the University of Wyoming.

Dr. E. M. BERRY, of Purdue University, has been appointed head of the department of mathematics at Lynchburg College.

Mr. E. C. BOWER, of the United States Naval Observatory, has been appointed assistant professor of mathematics and astronomy at Ohio Wesleyan University.

Associate Professor ABRAHAM COHEN, of Johns Hopkins University, has been promoted to a full professorship of mathematics.

Associate Professor H. B. CURTIS, of Marquette University, has been appointed head of the department of mathematics at Lake Forest College.

Assistant Professor H. G. H. GREENLEAF of De Pauw University has been appointed associate professor of mathematics.

Mr. D. C. HARKIN has been appointed lecturer in mathematics at the University of Manitoba.

At Cornell University, Professor W. A. HURWITZ has been made chairman of the department of mathematics.

Miss MYRA I. JOHNSON has been appointed professor of mathematics at Blackstone College for Women.

Mr. G. A. LYLE, of Lehigh University, has been appointed adjunct professor of mathematics at the Texas Technological College.

Associate Professor H. R. PHALEN, of the Armour Institute of Technology, has been appointed professor of mathematics at St. Stephen's College.

Assistant Professor G. C. PRIESTER of the University of Minnesota has been appointed associate professor of mathematics and mechanics and placed in charge of the materials testing laboratory in the college of engineering.

At the University of Wyoming, Professor O. H. RECHARD has been made head of the department of mathematics.

Acting Professor B. H. REDDITT of Lebanon Valley College has been appointed assistant professor of mathematics at Kenyon College.

Professor L. V. ROBINSON, of Oklahoma City University, has been appointed adjunct professor of mathematics at the Texas Technological College.

Professor S. A. ROWLAND, of Union College, has been appointed professor of mathematics at Ohio Wesleyan University.

Associate Professor MARY E. SINCLAIR, of Oberlin College, has been promoted to a full professorship of mathematics.

Assistant Professor H. L. SMITH, of the University of Minnesota, has been appointed assistant professor of mathematics at Louisiana State University.

Associate Professor F. W. SPARKS, of Louisiana State Normal College, has been appointed associate professor of mathematics at the Texas Technological College.

Assistant Professor J. S. TURNER, of Iowa State College, has been promoted to an associate professorship of mathematics.

Mr. A. H. WAIT, of the University of Wisconsin, has been appointed adjunct professor of mathematics at the Texas Technological College.

The following appointments as instructor in mathematics are announced:

Case School of Applied Science, Mr. RICHARD BURINGTON, Cornell University, Mr. A. G. MONTGOMERY and Mr. H. L. SCHUG; Hunter College, Miss LAURA GUGGENBUHL; Lehigh University, Mr. L. J. PARADISO and Mr. G. W. RIDDLE; Montana State School of Mines, Mr. ALEXANDER MASLOW; University of Nebraska, Mr. H. P. DOOLE; University of New Hampshire, Mr. M. R. SOLT; New York University (Washington Square College), Miss ELIZABETH BERGER; Princeton University, Mr. M. S. KNEBELMAN; Texas Technological College, Mr. P. K. REES.

Dr. A. HOWE, dean of the College of Liberal Arts of the University of Denver, and director of the Chamberlin Observatory of the University has died, at the age of sixty-seven.

Professor W. J. HUSSEY, director of the observatory of the University of Michigan, died in London, October 28, 1926, at the age of sixty-four. Professor Hussey was en route to Bloemfontein, South Africa, to assist in the establishment of a university observatory station there.

Dr. E. NIPHER, Professor emeritus of physics at Washington University, died October 6, 1926, at the age of seventy-eight.

Dr. C. A. WALDO, professor emeritus at Washington University, died October 1, 1926 at the age of seventy-four.

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ADDENDA AND CORRIGENDA

P. 44, middle, *for* "Dension" *read* "Denison".
P. 53, line 8 from bottom, *for* "Eurpoe" *read* "Europe".
P. 56, 1.2, *for* "Schloring" *read* "Schorling".
P. 121, in the determinant, insert "j" in first row, second column.
P. 123, formula (11) should read "$\nabla\cdot\sigma\times\tau=\tau\cdot\nabla\times\sigma-\sigma\cdot\nabla\times\tau$".
P. 123, in the display between (11) and (12) delete the second dot in "$\tau\cdot\sigma\cdot\times\tau=0$".
P. 123, last line, insert "f" as the third last denominator.
P. 217, insert "IV." preceding "A CRYSTALLOGRAPHIC . . . .".
P. 218, insert "V." preceding "ON THE DUALS . . . .".
P. 236, delete fifth and sixth last lines.
P. 255, Theorem III, for "paramteers" read "parameters".
P. 389, *for* problem number **3145** *read* **3149**. (See page 483).
P. 403, 1.20, *for* "molar" *read* "molecular".
P. 430, 1.6, et seq., Correction of problem numbers **3144 - 3193**.
P. 434, 1.4, *for* "Goast" *read* "Coast".
P. 459, last line, *for* "substracts" *read* "subtracts".
P. 478, title, *for* "UNDERGRADUATES" *read* "UNDERGRADUATE".
P. 487, title, *for* "847" *read* "487".
P. 488, 1.5 from bottom, *for* "are" *read* "is".
P. 498, title, insert "I" in "STAT STICS".
P. 499, 1.5, *for* "are" *read* "is".
P. 508, 1.2 from bottom, insert "a" after "perform".

Attention is called to the corrigendum on page 430 concerning the numbering of problems proposed and solved.

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IOWA, Cedar Rapids, April.

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KENTUCKY, Berea College, May 1.

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MARYLAND - DISTRICT OF COLUMBIA - VIRGINIA, Annapolis, Md., December 4.

MICHIGAN, Ann Arbor, Mich., April 28.

MINNESOTA, Northfield, Minn., May 22.

MISSOURI, Kansas City, Mo., November.

NEBRASKA, Bethany, Neb., May.

OHIO, Columbus, Ohio, April 2.

ROCKY MOUNTAIN, Colorado College, April, 1927.

SOUTHEASTERN, Atlanta, Ga., March 19-20.

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TEXAS, November.

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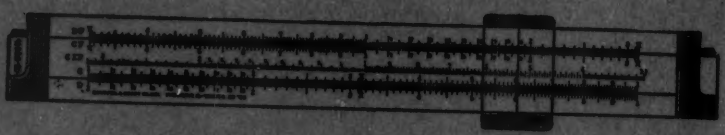

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